**Behavioural Economics** | **University of Oxford**
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**Practice Problems on Behavioural Game Theory**

In some of the empirical-judgment parts of questions more than one answer is defensible.

1. (a) Imagine that you are playing the following game with one other person, randomly selected from people who have not studied game theory. What is your choice: Stag or Rabbit? Explain your argument clearly, using whatever concepts you find helpful.

| | Stag | Rabbit |
|---|---|---|
| Stag | 2, 2 | 0, 1 |
| Rabbit | 1, 0 | 1, 1 |

(b) Now imagine you are playing the following game with twenty other people, randomly selected from people who have not studied game theory. What is your choice: Stag or Rabbit? Explain your argument clearly, using whatever concepts you find helpful.

2. Consider an Intersection game, in which two drivers meet at the intersection of two roads, with one on each road and no way to distinguish between their roles. The payoffs are:

| | Stop | Go |
|---|---|---|
| Stop | 0, 0 | 1, 1 |
| Go | 1, 1 | 0, 0 |

(a) Compute the mixed-strategy Nash equilibrium.

(b) Would you expect this game to yield the drivers Pareto-efficient payoffs if they have no way to distinguish between their roles? Explain.

Now imagine that a stoplight is installed at the intersection, which both players can see before they decide whether to Go or Stop. The stoplight is Green for one driver if it is Red for the other, and is equally likely to be Green for Row and Red for Column or the reverse when they meet.

(c) Show, in a new payoff matrix, how the stoplight changes the game and its set of equilibria. (Hint: The payoffs for the various outcomes are still as in the above matrix, but now players have more strategies because they can make their decision to Go or Stop depend on whether the light is Red or Green. To evaluate the consequences of their strategies, you will now have to make expected-payoff calculations that take into account their uncertainty about whether the light will be Green or Red for them and the effect this has on the final outcome.)

(d) Would you expect this game to yield players Pareto-efficient payoffs if they have no way to distinguish between their roles? Explain.

Now suppose that many drivers, in a large population, repeatedly, randomly meet in pairs to play the 2x2 Intersection game in part (a).

(e) What aggregate pattern or patterns (population frequencies) of Stop and Go decisions would you expect to see emerge in this population in the long run? Explain.

Now assume that a stoplight is installed at the intersection as in part (c).

(f) What aggregate pattern or patterns (that is, what population frequencies) of Stop and Go decisions would you expect to see emerge in this population in the long run? Explain.

3. In the following environments, a large number of identical players choose simultaneously between two pure strategies; they cannot randomize. In each case, graph the payoffs of the two strategies against the population frequency of the first strategy in a way that is consistent with the verbal description. Then use your graph to determine what pattern (or patterns) of behavior will emerge in the long run, and whether the pattern(s) that emerge(s) will be Pareto-efficient, in the sense of maximizing all players' average expected payoff.

(a) Each person can either install a car alarm in his car or not. Car alarms are highly effective when only a few cars have them, but (because people ignore them when they hear them go off too often) they are ineffective when most of the cars have them.

(b) There is a wall running through the center of your city, left over from the Cold War. Each person can either try to tear down the wall or ignore it. Everyone hates the wall, but everyone knows that if only a few people try to tear it down the government will arrest them and send them to jail. However, everyone also knows that if more than a few people try to tear it down, the government is unlikely to punish them.

(c) Each person can either shirk (effort level 1) or work hard (effort level 2). Each wishes to minimize the distance between his own effort level and the average effort level in the population (in other words, his payoff is minus this distance).

(d) Answer part (c) again, but assume that each person wishes to minimize the difference between his own effort level and one-half the average effort level in the population.

4. Suppose that the speed limit is 70 on the motorway, and that a large number N of drivers simultaneously and independently choose speeds from 70 to 100. Everyone prefers to go as fast as possible, other things equal, but the police are sure to ticket any driver whose speed is strictly faster than x% of the drivers, where x is a parameter such that $0 < x < 100$. (Thus, only by driving exactly 70 can a driver be sure of not being ticketed.) Suppose further that each driver ignores his own influence on the percentage, and the cost of being ticketed outweighs any benefit of going faster.

(a) Model this situation as a noncooperative game and analyze its set of pure-strategy Nash equilibria as far as possible. (Assume that x is a multiple of 1/N, that is $x = k/N$, where k is an integer.)

(b) Does the set of Nash equilibria depend on x when $0 < x < 100$?

(c) What is the set of pure-strategy Nash equilibria when the police don't ticket anyone? Explain.

(d) What is the set of pure-strategy Nash equilibria when the police ticket everyone who speeds? Explain.

(e) If the same drivers play this game repeatedly, observing the outcome after each play, how would you expect their speeds to change over time as they learn to predict each other's speeds? Explain intuitively or formally, whichever you prefer.

5. You and your sister (both risk-neutral expected-money maximizers) find two $1 bills on the sidewalk. Mom says that you can keep them if you can agree on how to divide them. There are only three possible ways to divide them: $2 to you, $0 to Sis; $1 to each of you; and $0 to you, $2 to Sis. Mom asks you to propose a division, which Sis can observe before deciding whether to say Yes or No. If Sis says Yes, Mom will enforce your proposal as the division, but if Sis says No, you both get nothing!

(a) Draw the game tree for this game, identifying your pure strategies by the amount you propose for yourself, $2, $1, or $0; and identifying Sis's pure strategies by specifying whether she says Y (for Yes) or N (for No) in each of the possible contingencies. (Remember that she gets to hear your proposal before deciding whether to say Yes or No.) In assigning payoffs, assume that both of you are expected-money-payoff maximizers.

(b) Draw the payoff matrix for this game, identifying pure strategies as in part (a).

(c) Identify all of the pure-strategy Nash equilibria in this game.

(d) Identify all of the subgame-perfect pure-strategy Nash equilibria in this game.

(e) Which strategy would you play? Explain your reasoning.

Now suppose that Mom asks you and Sis to submit simultaneous proposals (identified, for each of you, by the amount you propose to give yourself), with the understanding that if your proposals total $2 or less she will give each of you the amount you proposed, but if they total more than $2, you both get nothing.

(f) Answer part (a) again.

(g) Answer part (b) again.

(h) Answer part (c) again.

(i) Answer part (d) again.

(h) Answer part (e) again.

6. Two risk-neutral, expected money-maximizing bargainers, U and V, must agree on how to share $1. They bargain by making simultaneous demands; if their demands add up to more than $1, they each get nothing; if they add up to less than or equal to $1, each bargainer gets exactly his demand. Assume that any real number is a possible demand, and is also a possible division of the money.

(a) Find an infinite number of mixed-strategy Nash equilibria in this game. Explain why, in your equilibria, neither bargainer can do better by switching to any other strategy, pure or mixed.

(b) Show how to compute the equilibrium probability of disagreement, and show that it is always strictly positive in the mixed-strategy equilibria you identified in part (a).

(c) Are there any Pareto-efficient equilibria in this game?

(d) Now suppose that there are two plausible, but rival, notions of what it means to divide the dollar fairly. Redo your analysis from part (a), assuming that bargainers can put positive probability only on demands that are consistent with one or both of these notions of fairness. Is the equilibrium identified here also an equilibrium in the original game?

(e) Give a fairly detailed real-world (but not experimental) example in which common ideas of fairness appear to determine bargaining outcomes (and the likelihood of impasse) as in your answer to (d).

7. In each of the following games, identify the rationalizable strategies for each player, and then identify the equilibrium strategy combination or combinations. Then pick one rationalizable strategy for the Row player—a nonequilibrium strategy if this is possible in the game you are considering—construct beliefs that are consistent with common knowledge of rationality that support it as a best response, and explain your answer. What distinguishes the beliefs that support rationalizable strategies that are in equilibrium from those that support rationalizable strategies that are not in equilibrium?

(a)

| | L | C | R |
|---|---|---|---|
| T | 0<br>7 | 5<br>0 | 3<br>0 |
| M | 0<br>5 | 2<br>2 | 0<br>5 |
| B | 7<br>0 | 5<br>0 | 3<br>7 |

(b)

| | L | C | R |
|---|---|---|---|
| T | 0<br>7 | 5<br>0 | 7<br>0 |
| M | 0<br>5 | 2<br>2 | 0<br>5 |
| B | 7<br>0 | 5<br>0 | 0<br>7 |

8. Consider the following two-person guessing game. Each player has her/his own target, lower limit, and upper limit. These are possibly different across players, and they influence players' payoffs as follows. Players make simultaneous guesses, which are required to be within their limits. Each player then earns 1000 points minus the distance between her/his guess and the product of her/his target times the other's guess.

(a-d) Find the Nash equilibrium or equilibria for the following targets and limits:

a)

| | Lower Limit | Target | Upper Limit |
|---|---|---|---|
| Player 1 | 200 | 0.7 | 600 |
| Player 2 | 400 | 1.5 | 700 |

b)

| | Lower Limit | Target | Upper Limit |
|---|---|---|---|
| Player 1 | 300 | 0.7 | 500 |
| Player 2 | 400 | 1.3 | 900 |

c)

| | Lower Limit | Target | Upper Limit |
|---|---|---|---|
| Player 1 | 400 | 0.5 | 900 |
| Player 2 | 300 | 0.7 | 900 |

d)

| | Lower Limit | Target | Upper Limit |
|---|---|---|---|
| Player 1 | 300 | 1.3 | 500 |
| Player 2 | 200 | 1.5 | 900 |

(e) State and prove a general result that determines the equilibrium as a function of the targets and limits for these guessing games.

(f) Would you expect people randomly paired from students who have not studied game theory to play their equilibrium strategies in these guessing games? Explain why or why not. If not, explain what you think they might do instead.

9. Suppose three identical, risk-neutral firms must decide simultaneously and irreversibly whether to enter a new market which can accommodate only two of them. If all three firms enter, all get payoff 0; otherwise, entrants get 9 and firms that stay out get 8.

(a) Identify the unique mixed-strategy equilibrium and describe the resulting probability distribution of the total ex post number of entrants. (You are not asked to show this, but the game also has three pure-strategy equilibria, in each of which exactly two firms enter; but these equilibria are arguably unattainable in a one-shot game in the absence of prior agreement or precedent. The mixed-strategy equilibrium is symmetric, hence attainable.)

Now suppose that each firm follows a behavioral rule that is an independent and identically distributed draw from a distribution that assigns equal probabilities to two types: either *L1* (best response assuming the other firms are each equally likely to enter or stay out, and probabilistically independent), or *L2* (best response to *L1*).

(b) Describe the decisions of types *L1* and *L2* and the resulting *actual* (as opposed to what *L1* or *L2* expect) probability distribution of the total ex post number of entrants when each firm's type is drawn as explained above. Show that the expected number of entrants is closer to the ex post optimal number (2) than in your equilibrium from part (a), and that that the probability of exactly 2 entrants is higher than in (a). (In experiments subjects' initial responses come systematically closer to ex post optimality than the symmetric mixed-strategy equilibrium predicts, a result Kahneman has described as "magic." This analysis sh ows that bounded strategic rationality works like fairy dust.)

Now suppose that each firm follows a rule that is an independent and identically distributed draw from a distribution that assigns probability ½ to type *L1*, ¼ to*L2*, and ¼ to a type called *Sophisticated*, which plays an equilibrium in the game in which the prior probabilities of *L1*, *L2*, and *Sophisticated* players are common knowledge.

(c) Plugging in the behaviors of *L1* and *L2* players (which do not depend on the prior type probabilities), characterize equilibrium in the game played by *Sophisticated* players.

(d) How does your answer to (c) change, if at all, if the prior probability of *Sophisticated* players is ε ≈ 0, and the prior probability of *L2* players is ½ -ε (with the prior probability of *L1* players held constant at ½)?

10. (a) Imagine that you are playing the following game as Row player, with one other person, randomly selected from people who have not studied game theory. What is your choice: H or T? Explain your argument clearly, using whatever concepts you find helpful.

| | H | T |
|---|---|---|
| H | -2<br>2 | 1<br>-1 |
| T | 1<br>-1 | -1<br>1 |

(b) Now imagine that you are playing the game as Column player, with one other person, randomly selected from people who have not studied game theory. What is your choice: H or T? Explain your argument clearly, using whatever concepts you find helpful.

11. Consider a two-person game with payoff matrix as shown. Before choosing simultaneously between T and B, or L and R, Column must send R a costless, nonbinding (“cheap talk”) message announcing her/his intention to play either L or R. Both players know the rules of the game, including the values of *x* and y, as common knowledge.

| | L | R |
|---|---|---|
| T | 2<br>2 | x<br>0 |
| B | 1<br>1 | y<br>1 |

(a) For what values of x and y are the choices T for Row and L for Column (each with probability one) consistent with subgame-perfect equilibrium in the entire game?

(b) For what values of x and y are the choices T for Row and L for Column (each with probability one) each part of some rationalizable strategy (in the entire game)?

In “Nash Equilibria are not Self-Enforcing” (in *Economic Decision Making: Games, Econometrics and Optimisation*, edited by Gabszewicz, Richard, and Wolsey, Elsevier 1990) Aumann argues that in games like this with $x \geq y$, an announcement by Column that s/he intends to play L should not (or will not, in a positive theory) alter Row’s belief that Column will actually play L, because Column does as well or better when Row plays T without regard to whether Column plays L or R.

(c) Do either subgame-perfect equilibrium or rationalizability distinguish between the credibility of an announcement by Column that s/he intends to play L when $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$?

(d) What assumptions about strategic behavior suffice to justify Aumann’s argument against the credibility of such an announcement.

(e) Evaluate the credibility of an announcement by Column that s/he intends to play L behaviorally, making whatever assumptions and using whatever arguments and evidence you find useful. What, if any, meanings might such an announcement convey beyond those it conveys in arguments based on subgame-perfect equilibrium or rationalizability? Make clear how and why your evaluation of the credibility of the announcement distinguishes between games where $2 \geq x > y$, $2 \geq x = y$, or $2 \geq y > x$.

12. Consider the Battle of the Sexes game. Assume, here and below, that the structure is common knowledge, that both players are self-interested, and that there are no observable differences between the players or their roles in the game. In each of the variations described below, say whether you would expect the players to be able to coordinate on one of the efficient pure-strategy equilibria, and what strategies you would expect the players to use, on average. Briefly but clearly explain your answers.

| | Fights | Ballet |
|---|---|---|
| Fights | 1<br>3 | 0<br>0 |
| Ballet | 0<br>0 | 3<br>1 |

Battle of the Sexes

(a) The original simultaneous-move game is a complete model of the players' situation.

(b) The game is modified so that Row chooses her/his strategy first and Column gets to observe her/his choice before choosing her/his own strategy.

(c) Row chooses her/his strategy first and Column does NOT get to observe her/his choice before choosing her/his own strategy.

(d) Row chooses her/his strategy first, Column observes her/his choice before choosing her/his own strategy, but Row then gets to observe Column’s choice and costlessly revise her/his own choice, and this decision ends the game (so that Column cannot revise her/his choice).

(e) The original simultaneous-move game is a complete model of the players' situation, except that Row (only) can make a non-binding suggestion about the strategies players should use before they choose them.

(f) The original simultaneous-move game is a complete model of the players' situation, except that both players can make simultaneous, non-binding suggestions about the strategies players should use before they choose them.

(g) The original simultaneous-move game is a complete model of the players' situation, except that players can make sequential, non-binding suggestions about the strategies players should use before they choose them, say with Row making the first suggestion.

13. Consider the following two-person zero-sum betting game with private information. Each of two players, 1 and 2, is given information about which of three ex ante equally likely states has occurred, A, B, or C. As indicated by the borders in the table, player 1 learns either that the state is {A or B} or C; player 2 learns either that the state is A or {B or C}. Once informed, the players choose simultaneously between two decisions: Bet or Pass. A player who chooses Pass earns 10 no matter what the state. If and only if both players choose Bet, they get the payoffs in the table, depending on which state has occurred. (If one chooses Bet while the other chooses Pass, they both earn 10.) Assume that the rules of the game and the information structure are common knowledge, as is players' rationality.

| player/state | A | B | C |
|---|---|---|---|
| 1 | 25 | 5 | 20 |
| 2 | 0 | 30 | 5 |

(a) Identify the (Bayesian) Nash equilibrium in this game (which is unique). Can betting ever take place in equilibrium? Explain.

(b) Is the game (weakly) dominance-solvable? If so, in how many rounds?

(c) In experiments with games like these and naïve subjects, approximately half of them bet. Sketch a theory of strategic behavior that has the potential to explain this, and use it to derive a refutable prediction about how subjects' betting patterns deviate from equilibrium.