MEANINGFUL THEOREMS: NONPARAMETRIC ANALYSIS OF REFERENCE-DEPENDENT PREFERENCES[0]

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Laura Blow
Department of Economics, University of Surrey
Ian Crawford
Department of Economics, University of Oxford; and Nuffield College, Oxford
Vincent P. Crawford
Department of Economics, University of Oxford; All Souls College, Oxford; and Department of Economics, University of California, San Diego

Abstract: We derive nonparametric sufficient and asymptotically necessary conditions for the existence of reference-dependent preferences, as in Kőszegi and Rabin's (2006) structural implementation of Kahneman and Tversky's (1979) prospect theory, that can rationalize some consumer demand choices that violate the conditions for a neoclassical rationalization. Our conditions relax Kőszegi and Rabin's and others' assumption of additive separability across goods. Unless prospect theory's notion of sensitivity is constant and reference points are precisely modelable or observable, the hypothesis of reference-dependent preferences has few or no nonparametrically refutable implications. But with constant sensitivity and modelable reference points, the model has useful implications. For almost all of Farber's (2005, 2008) cabdrivers, models that relax additive separability across goods have higher measures of predictive success (Selten and Krischker 1983) than their additively separable counterparts. For roughly half of the drivers, reference-dependent models that relax additive separability have higher Selten measures than their neoclassical counterparts.

Keywords: revealed preference, nonparametric demand analysis, prospect theory, reference-dependent preferences, loss aversion, consumer theory, labor supply

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Laura Blow: l.blow@surrey.ac.uk
Ian Crawford: ian.crawford@economics.ox.ac.uk
Vincent P. Crawford: vincent.crawford@economics.ox.ac.uk

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1. INTRODUCTION

In their analysis of prospect theory, Kahneman and Tversky (1979) (also Tversky and Kahneman 1991) introduce a model in which people have preferences over gains and losses in consumption relative to a reference point. Such reference-dependence alters the domain of preferences from levels of consumption, as in neoclassical consumer theory, to changes in it, but it is not inherently inconsistent with a complete and transitive preference ordering over changes. Although Kahneman and Tversky focus on changes in consumption, Kőszegi and Rabin (2006) and most modern analyses allow preferences to respond to levels of consumption as well as to changes; and preferences over both levels and changes are again not inherently irrational.

Reference-dependent models have played an important role in empirical analyses of workers', consumers', and investors' choice behavior since Camerer, Babcock, Loewenstein, and Thaler's (1997) analysis of the daily labor supply of New York City cabdrivers.[1] A neoclassical model of labor supply, analogous to a model of consumer demand for earnings and leisure, predicts a positive elasticity of hours worked with respect to the wage unless there are very large income effects. However, Camerer et al., taking expected earnings per hour as analogous to the wage, estimate a strongly negative elasticity. To explain it they propose a model in which drivers have daily earnings targets, analogous to Kahneman and Tversky's reference points. Experiments suggest that most people are loss-averse—more sensitive to changes below their targets (losses) than above them (gains). Loss aversion creates a kink that makes a driver's daily earnings tend to bunch around his earnings target, thus working less on days with high "wages". This allows a reference-dependent model, in which drivers respond to changes in earnings as well as levels, to reconcile the negative earnings elasticity of hours worked with the positive incentive effect of increased expected earnings per hour.

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[1] Cabdrivers' labor supply is of particular interest empirically because, unlike most workers in modern economies, many choose their own hours. Another impetus to applications was Kahneman, Knetsch, and Thaler's (1990) experimental analysis of the endowment effect, whereby a person's willingness to accept money for a good he owns exceeds his willingness to pay for it. More recent applications include Oettinger's (1999) study of stadium vendors; Genesove and Mayer's (2001) study of home sellers; Fehr and Goette's (2007) field experiment on bicycle messengers; Post, van den Assem, Baltussen, and Thaler's (2008) analysis of the game show Deal or No Deal; and Pope and Schweitzer's (2011), Lien and Zheng's (2015), and Meng and Weng's (2018) field analyses of risky choice.

Although reference-dependent models allow a coherent rationality-based explanation of some choice behavior that is anomalous from a neoclassical point of view, two factors have limited their appeal. Because reference-dependent models expand the domain of preferences, some researchers doubt that they yield any testable implications—Samuelson's (1947) "meaningful theorems". Such doubts are exacerbated when reference points are not observed or modeled. Further, empirical implementations of reference-dependent models have so far relied on parametric structural assumptions that go well beyond theory or evidence.[2] As McFadden (1985) remarks, using econometrics to flesh out the theory in this way "interposes an untidy veil between econometric analysis and the propositions of economic theory."[3]

This paper addresses these issues in a nonparametric analysis of reference-dependent consumer theory. In the neoclassical case where preferences respond only to levels of consumption, Afriat (1967), Diewert (1973), and Varian (1982) show, in the revealed-preference tradition of Samuelson (1948) and Houthakker (1950), that a price-taking consumer's demand behavior can be nonparametrically rationalized by the maximization of a nonsatiated utility function if and only if the data satisfy the Generalized Axiom of Revealed Preference ("GARP"; Definition 3, Section 4.1). The benefits of a nonparametric approach are well understood. The theory's testable implications then consist of a set of inequality restrictions on the observable, finite data, rather than of shape restrictions on objects that are not directly observable such as indifference curves, demand curves, or labor supply curves. Those implications can be checked directly, without estimating econometric models of unobservable objects. The theory also largely avoids the need for the auxiliary statistical assumptions that structural econometric approaches need for consistent estimation.[4]

Recent structural econometric work on reference-dependent models of labor supply sets the stage for our analysis. Farber (2005, 2008) analyzes a panel dataset on New

[2] Farber (2005, 2008, 2015), Post et al. (2008), Crawford and Meng (2011), Pope and Schweitzer (2011), Gill and Prowse (2012), Doran (2014), Lien and Zheng (2015), and Meng and Weng (2018) reduce reliance on parametric assumptions in various ways; but none of them eliminate such reliance.
[3] McFadden's remark refers mainly to parametric models; but even nonparametric methods require assumptions about the structure and the distributions of unobservable variables, which assumptions are seldom if ever well motivated by theory.
[4] Measurement error is an exception to such immunity, but it too can be accommodated nonparametrically (Varian 1985).

York City cabdrivers, allowing earnings targeting.[5] In Farber's data, as in Camerer et al.'s (1997), hours worked and earnings per hour are strongly negatively correlated overall. Farber finds that a reference-dependent model with daily earnings targets, treated as latent variables, fits better than a neoclassical model. But his estimates of the earnings targets are unstable, which he argues precludes a useful reference-dependent model. Farber (2015) analyzes a newer and much larger dataset on New York City cabdrivers and finds that "…drivers tend to respond positively to unanticipated as well as anticipated increases in earnings opportunities", as in a reference-dependent model like those studied here. But he again concludes that "…consideration of gain-loss utility and income reference-dependence is not an important factor in the daily labor supply decisions of taxi drivers."

In a theory paper inspired by Camerer et al.'s and Farber's (2005) analyses, Kőszegi and Rabin (2006) adapt Kahneman and Tversky's model of reference-dependent preferences to economic applications. Kőszegi and Rabin allow preferences to reflect additively separable components of neoclassical consumption utility and reference-dependent "gain-loss" utility. Unlike Kahneman and Tversky, who take no definite position on how reference points are determined, Kőszegi and Rabin close their model by setting an agent's reference point equal to the rational expectation of consumption, good by good. Their model reconciles the neoclassical intuition that higher wages tend to increase drivers' labor supply with a negative overall correlation between hours worked and expected earnings per hour: For perfectly anticipated changes in earnings, or hours, gain-loss utility drops out of the model, which then reflects the neoclassical intuition. But for unanticipated changes loss aversion makes choices bunch around the reference point, which can yield a negative overall correlation.

Crawford and Meng (2011) adapt Kőszegi and Rabin's (2006) model to reconsider Farber's (2005, 2008) econometric analyses, using Farber's original dataset. Allowing

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[5] As Farber (2008, p. 1069) suggests, taking reference-dependence into account is essential to correctly identify even the determinants of behavior that don't directly involve reference-dependence: "Evaluation of much government policy regarding tax and transfer programs depends on having reliable estimates of the sensitivity of labor supply to wage rates and income levels. To the extent that individuals' levels of labor supply are the result of optimization with reference-dependent preferences, the usual estimates of wage and income elasticities are likely to be misleading."

drivers to have daily reference points for hours and earnings, Crawford and Meng use natural sample proxies to model Kőszegi and Rabin's rational-expectations reference points, making it unnecessary to treat them as latent variables and avoiding the instability that led Farber to doubt the usefulness of a reference-dependent model.[6] Crawford and Meng's analysis suggests that a reference-dependent model can give a useful account of Farber's cabdrivers' behavior.

Like almost all empirical applications of reference-dependent models to date, Farber's (2005, 2008) and Crawford and Meng's (2011) analyses rely on parametric assumptions about functional structure and form that are not directly supported by theory or evidence. Both assume, like Kőszegi and Rabin (2006), that preferences are additively separable across consumption and gain-loss utility, and that consumption and gain-loss utility are additively separable across goods. Crawford and Meng also assume, like Kőszegi and Rabin, that gain-loss utility is determined good by good by the difference between realized and reference consumption utilities.

Finally, Farber and Crawford and Meng assume, as Kőszegi and Rabin often do (their assumption A3′), that reference-dependent preferences have constant sensitivity (Tversky and Kahneman's 1991 "sign-dependence"). Specifying a reference point divides commodity space into gain-loss regimes, such as "earnings-loss and hours-gain" in the model of labor supply. Constant sensitivity requires that preferences over consumption bundles are the same for all bundles in a given regime, but leaves preferences free to vary across regimes. We call the general case, in which constant sensitivity is not imposed, variable sensitivity.

We derive nonparametric conditions, in the spirit of Afriat's (1967), Diewert's (1973), and Varian's (1982) conditions for the neoclassical case, for the existence of reference-dependent preferences over changes in and levels of consumption that rationalize a price-taking consumer's demand behavior. Our model follows Kőszegi and Rabin's, but without directly imposing their assumptions on functional structure or Farber's and Crawford and Meng's assumptions on functional structure and form.

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[6] Crawford and Meng also take advantage of sampling variation to simplify Kőszegi and Rabin's probabilistic targets to point expectations. Kőszegi and Rabin make no allowance for errors, so that only nondegenerate target distributions can create the deviations from expectations that allow expectations-based reference-dependence to have any effect.

Our goal is to learn to what extent the refutable implications of reference-dependent models of consumer demand stem from reference-dependence, constant sensitivity, or loss aversion per se, or are artifacts of the ancillary assumptions in previous analyses.[7]

We illustrate our methods by re-analyzing Farber's data, asking whether reference-dependent models that relax the ancillary assumptions on functional structure and form allow better explanations of drivers' choices.

Our analysis builds on Afriat's, Diewert's, and Varian's nonparametric analyses of neoclassical consumer demand, which rely essentially on the rationality assumption. Although our analysis covers cases where a neoclassical rationalization is impossible, we can adapt their rationality-based methods because we expand the domain of preferences, in the disciplined way suggested by reference-dependence.[8] Our analysis raises issues beyond those addressed in previous nonparametric analyses because a consumer chooses levels of and changes in consumption bundled and priced together, and his choices can influence reference-dependent preferences by changing how consumption relates to the reference point.

Our analysis shows that reference-dependent preferences can make it possible to rationalize some choice behavior that violates GARP, depending on two factors: (i) whether sensitivity is constant as Farber (2005, 2008), Crawford and Meng (2011), and sometimes Kőszegi and Rabin (2006) assume, or variable; and (ii) whether reference points are unobservable and unmodelable as Camerer et al. (1997) and Farber assume, or observable or precisely modelable (henceforth "modelable") as Kőszegi and Rabin and Crawford and Meng assume. Our results for variable sensitivity do not depend on how it varies. And our conditions are independent of the interpretation of reference points, which need not be expectations.

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[7] To our knowledge ours is the first nonparametric analysis of a reference-dependent model, except for Gul and Pesendorfer (2006), Abdellaoui, Bleichrodt, and Paraschiv (2007), Ok, Ortoleva, and Riella (2015), and Freeman (2019), which focus on different aspects of the problem. Experimental studies like Kahneman et al. (1990); Abeler, Falk, Goette, and Huffman (2011); and Gill and Prowse (2012) reveal aspects of reference-dependence that choices from linear budget sets cannot, raising further issues. Nonparametric analyses of other "behavioral" issues include Crawford (2010) on habit formation and Fang and Wang (2015) and Blow, Browning, and Crawford (2021) on intertemporal choice.

[8] By contrast, Farber (2008, p. 1070) focuses on preferences that respond only to levels of consumption and concludes by implication that most of his drivers are irrational: "This [earnings-targeting] is clearly nonoptimal from a neoclassical perspective, since it implies quitting early on days when it is easy to make money and working longer on days when it is harder to make money. Utility would be higher by allocating time in precisely the opposite manner."

Section 2 introduces our model of reference-dependent preferences. We characterize preferences that have additively separable consumption and gain-loss utility components, as in Kőszegi and Rabin (2006), and that satisfy constant sensitivity and a natural assumption of joint continuity in the consumption bundle and reference point. This yields a class of preferences that nest Farber's, Kőszegi and Rabin's, and Crawford and Meng's models with constant sensitivity. The class allows the preferences over consumption bundles induced by consumption plus gain-loss utility to vary freely across gain-loss regimes, with no need for marginal rates of substitution to satisfy knife-edge cross-regime restrictions as they do in Kőszegi and Rabin's model under their assumption A3′ (see Crawford and Meng's Table 1). Our characterization also relaxes Farber's, Kőszegi and Rabin's, and Crawford and Meng's assumption that the induced preferences over bundles are additively separable across goods, which is important in our re-analysis of Farber's data.

Section 3 considers rationalization when reference points are unobservable and unmodelable. In that case we assume that a reference point can be chosen hypothetically for each observation as part of a rationalization—nonparametrically paralleling Camerer et al.'s (1997) and Farber's (2005, 2008) structural treatments of earnings targets as latent variables. Whether sensitivity is constant or variable, hypothesizing reference points gives reference-dependent preferences enough flexibility to rationalize any data, making the model nonparametrically irrefutable. The rationalization in our proof satisfies Farber's, Kőszegi and Rabin's, and Crawford and Meng's functional structure and form assumptions, so they too are nonparametrically untestable. Put another way, the refutable implications of reference-dependent preferences in analyses that treat targets as latent variables are largely a by-product of functional structure and form assumptions ("largely", not entirely, only because estimating latent targets somewhat limits their freedom).

Section 4 considers rationalization when reference points are modelable or observable. Then, with variable sensitivity, the only nonparametrically refutable implications of reference-dependent preferences are that any subset of observations that share exactly the same reference point must satisfy GARP, as a trivial implication of Afriat's Theorem. For such a subset, reference-dependent preferences reduce to neoclassical preferences, so that allowing reference-dependence adds nothing empirically useful to the neoclassical model.

Section 4 also considers rationalization when reference points are modelable and preferences have constant sensitivity. The model then has nonparametric implications that are empirically useful. Recall that a reference point divides commodity space into gain-loss regimes, within each of which, with constant sensitivity, the induced preferences over consumption bundles must be the same. A rationalization plainly requires that each regime's observations satisfy GARP, by Afriat's Theorem, in which case there exist preferences that rationalize each regime's data within the regime. With finite data there is normally a range of rationalizing regime preferences (Varian 1982, Fact 4). In addition, with constant sensitivity a rationalization requires that there is some combination of rationalizing regime preferences such that the consumer cannot gain by "defecting" from any observation's consumption bundle to some bundle in its budget set in another regime (which is potentially beneficial depending on how changing regimes alters his preferences). Together those sets of conditions are necessary and sufficient for a reference-dependent rationalization.

Those conditions are not directly applicable because they are conditional on the choices of rationalizing regime preferences, which involve complex trade-offs. We address this difficulty by showing that GARP for each regime's observations plus no-defection conditions based on the "Afriat" rationalizing regime preferences from the proof of Afriat's Theorem (Section 4.1) are sufficient for a rationalization. The resulting conditions are directly applicable, but with finite data they are not necessary because they rely on the choice of the Afriat regime preferences. However, in the limit as the data become "rich", so each regime's range of convexified rationalizing regime preferences collapses on its Afriat regime preferences (Mas-Colell 1978 and Forges and Minelli 2009), those sufficient conditions are asymptotically necessary.

Our theoretical analysis identifies a grain of truth in the common belief that allowing reference-dependence destroys the parsimony of neoclassical consumer theory: Nonparametrically refutable implications depend on modeling reference points and restricting sensitivity—as is done in structural analyses, more flexibly, via functional structure and form assumptions. Even so, our main conclusion is positive, in that we identify a large class of models that can yield parsimonious, rationality-based explanations of demand behavior that violates the conditions for a neoclassical rationalization, without directly imposing functional structure or form assumptions.

The roles of modelability of reference points and constant sensitivity in our analysis are more important than one might guess: Much of the literature is silent on the importance of modelability (Kőszegi and Rabin 2006 and Crawford and Meng 2011 are exceptions) but it is necessary for a reference-dependent model to have *any* nonparametrically refutable implications. And the literature treats constant sensitivity as merely a convenient simplification, but it too is needed for useful implications.

Section 4's results suggest methods for recovering rationalizing reference-dependent preferences when they exist that are more complex than checking GARP gain-loss regime by regime. Because those methods build on Diewert's and Varian's linear-programming methods for the neoclassical case, they remain computationally tractable even in large datasets. Section 5 illustrates them by reconsidering Farber's (2005, 2008) and Crawford and Meng's (2011) structural analyses nonparametrically, using Farber's dataset. We follow Farber in assuming constant sensitivity and Crawford and Meng in assuming constant sensitivity and modelable reference points, as our results show is necessary for a useful nonparametric analysis. We deviate from their assumption that drivers have homogeneous preferences, as is often assumed in the labor supply literature, instead allowing unrestricted heterogeneity of drivers' preferences, as is usual in the nonparametric demand literature. (Our theoretical analysis covers both cases.) We also deviate from Crawford and Meng's analysis by comparing several models of reference points as alternatives to their simple sample proxies for Kőszegi and Rabin's rational-expectations reference points.

The GARP condition for a neoclassical rationalization is violated for most of Farber's drivers, but our analysis yields a coherent reference-dependent rationalization for almost all of those drivers' choices. It is of course unsurprising that reference-dependent models fit better, because they nest their neoclassical counterparts. The real question is whether their fits are sufficiently better to justify their greater flexibility. We address this question using Selten's measure of predictive success (Selten and Krischker 1983, Selten 1991, Beatty and Crawford 2011), which compares models' fits nonparametrically, controlling for their flexibility as measured by the likelihood that randomly generated data are consistent with the model.

For reference-dependent models the Selten measures can be computed only approximately because of the finite-sample gap between our sufficient and asymptotically necessary conditions for a rationalization, but we derive exact bounds for them. For almost all of Farber's drivers, models that relax the assumption maintained in all previous work that preferences are additively separable across goods strongly out-perform their counterparts that impose additive separability. Relaxing additive separability across goods, for each of the alternative reference-point models we consider, for roughly half of Farber's drivers a reference-dependent model does as well or unambiguously better (given the bounds) than the neoclassical model. For roughly another half, the neoclassical model does as well or better. (For a small minority of drivers the bounds are not tight enough for a clear comparison.) We conclude that for many of Farber's drivers, reference-dependent models of labor supply do provide a useful supplement to the neoclassical model.

Section 6 summarizes our analysis and conclusions.

2. REFERENCE-DEPENDENT PREFERENCES

We consider whether reference-dependent preferences can rationalize a finite set of consumer demand observations for a single consumer—or equivalently a group of consumers assumed to have homogeneous preferences as in Camerer et al. (1997), Farber (2005, 2008), and Crawford and Meng (2011), but we will speak of a single consumer. Index goods $k = 1,\ldots, K$. We assume the consumer is a price-taker, who chooses among consumption bundles $\boldsymbol{q} \in \mathbb{R}_+^K$ with linear budget constraints. His preferences are represented by a family of utility functions $u(\boldsymbol{q}, \boldsymbol{r})$, parameterized by an exogenous reference point $\boldsymbol{r} \in \mathbb{R}_+^K$, conformable to a K–good consumption bundle as in Tversky and Kahneman (1991) and Crawford and Meng (2011).

We index observations $t = 1,\ldots, T$. In Section 3's analysis of the case where reference points are unobservable and unmodelable, the data are prices and quantities $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ and the hypothetical reference points that arise as part of a rationalization in that case are denoted $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$. In Section 4's analysis of the case where reference points are observable or modelable, the data are prices, quantities, and reference points $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\ldots,T}$. The interpretation of $\boldsymbol{r}_t$ will be clear from the context. We sometimes denote goods as scalars indexed by subscripts, so for each good $k = 1,\ldots, K$, $\boldsymbol{q} \equiv (q^1, \ldots, q^K)$ and for each observation $t = 1,\ldots, T$, $\boldsymbol{q_t} \equiv (q_t^1, \ldots, q_t^K)$, with analogous notation for $\boldsymbol{p}$, $\boldsymbol{p_t}$, $\boldsymbol{r}$, and $\boldsymbol{r_t}$.

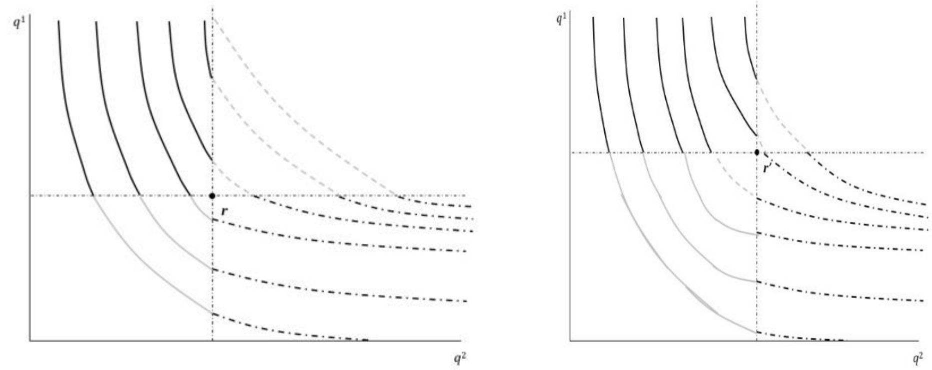
We assume that the consumer's utility function $u(\boldsymbol{q}, \boldsymbol{r})$ is strictly increasing in $\boldsymbol{q}$ and, except where noted, strictly decreasing in $\boldsymbol{r}$, to describe preferences that respond positively to changes in consumption relative to the reference point as well as to consumption levels. Under those assumptions our specification is as flexible as a general strictly increasing function of levels $\boldsymbol{q}$ and changes $\boldsymbol{q} - \boldsymbol{r}$. It nests the neoclassical case where preferences respond only to levels; Kahneman and Tversky's (1979) and Tversky and Kahneman's (1991) case where they respond only to changes; and cases like Farber's (2005, 2008), Kőszegi and Rabin's (2006), and Crawford and Meng's (2011) where preferences respond to both levels and changes. Like Tversky and Kahneman (1991), Kőszegi and Rabin (2006), Farber (2005, 2008), and Crawford and Meng (2011), except where noted we take $u(\boldsymbol{q}, \boldsymbol{r})$ to be jointly continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$. Although joint continuity of preferences is not testable with finite data, it is a plausible requirement, ensuring that in the limit as the data become rich in the sense defined in Section 4.4, the consumer's demand behavior does not vary unrealistically with small changes in the reference point.

We use the term variable sensitivity for the general case of preferences that can be represented by a utility function $u(\boldsymbol{q}, \boldsymbol{r})$ in the class just described. An important special case is that of constant sensitivity (Tversky and Kahneman's 1991 sign-dependence; Kőszegi and Rabin's 2006 assumption A3′). Let $\text{sign}(\boldsymbol{q} - \boldsymbol{r})$, the vector whose kth component is $\text{sign}(q^k - r^k)$, be the good-by-good sign pattern of gains and losses. A reference point divides commodity space into gain-loss regimes, throughout each of which $\text{sign}(\boldsymbol{q} - \boldsymbol{r})$ remains constant. With constant sensitivity the consumer's preferences over consumption bundles $\boldsymbol{q}$ must be the same for all observations within a regime and otherwise independent of $\boldsymbol{r}$; but his preferences are free to vary across regimes. In Tversky and Kahneman's (1991, p. 1046) words, preferences are characterized "not by a single preference order but by a family or a book of preference orders". Constant sensitivity rules out strong local variations in preference within regimes, which are theoretically possible but empirically implausible. Such strong variations are usually ruled out in structural models implicitly (and more flexibly) via functional form assumptions. But in a nonparametric analysis, constant sensitivity may be the only way to limit them.

> *DEFINITION 1: [Preferences and utility functions with constant sensitivity.] A reference-dependent utility function $u(\boldsymbol{q},\boldsymbol{r})$ satisfies constant sensitivity if and only if, for any bundles $\boldsymbol{q}$ and $\boldsymbol{q}^*$ and reference points $\boldsymbol{r}$ and $\boldsymbol{r}^*$ such that $sign(\boldsymbol{q}-\boldsymbol{r}) = sign(\boldsymbol{q}^*-\boldsymbol{r}) = sign(\boldsymbol{q}-\boldsymbol{r}^*) = sign(\boldsymbol{q}^*-\boldsymbol{r}^*)$, $u(\boldsymbol{q},\boldsymbol{r}) \geq u(\boldsymbol{q}^*,\boldsymbol{r})$ if and only if $u(\boldsymbol{q},\boldsymbol{r}^*) \geq u(\boldsymbol{q}^*,\boldsymbol{r}^*)$.*

With two goods, a reference point in the interior of commodity space divides it into four (2^2) gain-loss regimes.[9] Because $\boldsymbol{r}$ is unrestricted, each regime's preferences over consumption bundles $\boldsymbol{q}$ must be defined for the entire commodity space: Each value of sign$(\boldsymbol{q}-\boldsymbol{r})$ "switches on" a different regime's preferences. Figure 1's panels show four regime indifference maps and the associated global indifference maps for reference points $\boldsymbol{r}$ and then $\boldsymbol{r}'$. The shift from $\boldsymbol{r}$ to $\boldsymbol{r}'$ does not alter the regime maps, but as $\boldsymbol{r}$ varies, even locally, the changing regime boundaries alter how the regime maps connect, and thereby alter the global map.

Figure 1. A set of regime maps with constant sensitivity and the associated global map for alternative reference points



Following Kőszegi and Rabin (2006) we assume (as Definition 1 allows, but does not require) that preferences have consumption utility and gain-loss utility

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[9] Our analysis allows reference-dependence to be inactive on one or more dimensions, with some reference points on the boundary of commodity space as in Figures 3-5 below.

components that enter $u(\boldsymbol{q},\boldsymbol{r})$ additively separably, with the consumption utility function the same for all gain-loss regimes and independent of the reference point.

Proposition 1 characterizes preferences and associated utility functions $u(\boldsymbol{q},\boldsymbol{r})$ that satisfy Kőszegi and Rabin's assumption that preferences are additively separable across consumption and gain-loss utility; constant sensitivity; joint continuity; differentiability; and an assumption that $K \geq 2$, with reference-dependence active for all K goods, and that the induced preferences over consumption bundles have marginal rates of substitution that differ across gain-loss regimes throughout some open neighborhood of commodity space. Then, preferences must be representable by a utility function $u(\boldsymbol{q},\boldsymbol{r})$ with: (i) gain-loss utilities that are additively separable across regimes, across $\boldsymbol{q}$ and $\boldsymbol{r}$, and across goods within each regime; and (ii) gain-loss utilities whose good-by-good responses to reference points exactly mirror their responses to bundles.[10] Given $\{\boldsymbol{r}_t\}_{t=1,\dots,T}$, let $k = 1, \dots, K$ and let $\boldsymbol{g_t}$ be a vector of length K, with kth binary component 0 if $q_t^k < r_t^k$ and 1 if $q_t^k \geq r_t^k$. Let $I_g(\boldsymbol{q_t},\boldsymbol{r_t}) = 1$ if $\boldsymbol{q_t} \in regime\ g\ for\ \boldsymbol{r_t}$ and 0 otherwise.

PROPOSITION 1: [Preferences and utility functions with constant sensitivity and joint continuity.] Suppose that a reference-dependent preference ordering or associated utility function has additively separable consumption utility and gain-loss utility components; and that there are $K \geq 2$ goods, with reference-dependence active for all K goods. Then the ordering satisfies constant sensitivity if and only if an associated utility function can be written, for some consumption utility function $V(\cdot)$ and regime utility functions $V_g(\cdot,\cdot)$ and $v_g(\cdot)$, as

$$(1) \qquad u(\boldsymbol{q},\boldsymbol{r}) \equiv V(\boldsymbol{q}) + \textstyle\sum_g I_g(\boldsymbol{q},\boldsymbol{r}) V_g(v_g(\boldsymbol{q}),\boldsymbol{r}).$$

Suppose further that the preference ordering is differentiable and that the induced preferences over consumption bundles

[10] Conversely, any combination of well-behaved regime preferences is consistent with joint continuity for some gain-loss utilities. These conclusions hold, mutatis mutandis, if reference-dependence is active for two or more goods.

have marginal rates of substitution that differ across gain-loss regimes throughout some open neighborhood of commodity space. Then the preference ordering satisfies constant sensitivity and joint continuity if and only if there is an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ *that can be written, for some consumption utility function* $V(\cdot)$ *and regime component utility functions* $v_g^k(\cdot)$*, as*

(2) $$u(\boldsymbol{q},\boldsymbol{r}) \equiv V(\boldsymbol{q}) + \textstyle\sum_{g,k} I_g(\boldsymbol{q},\boldsymbol{r})[v_g^k(q^k) - v_g^k(r^k)].$$

PROOF: The "if" part of each claim is immediate. The "only if" part regarding (1) follows from Definition 1 via standard characterization results for additively separable preferences. To prove the "only if" part regarding (2), note that $u(\boldsymbol{q},\boldsymbol{r})$ in (1) satisfies joint continuity if and only if, for any regimes g and g' that differ in the gain-loss status of good i, and $\boldsymbol{q}$ and $\boldsymbol{r}$ such that $q^i = r^i$,

(3) $$V_g\big(v_g(\boldsymbol{q}),\boldsymbol{r}\big) = V_{g'}\big(v_{g'}(\boldsymbol{q}),\boldsymbol{r}\big).$$

We claim that (3) can hold for any $\boldsymbol{q}$, $\boldsymbol{r}$, and *i* such that $q^i = r^i$ only if $V_g\big(v_g(\boldsymbol{q}),\boldsymbol{r}\big)$ and $V_{g'}\big(v_{g'}(\boldsymbol{q}),\boldsymbol{r}\big)$ are additively separable in $\boldsymbol{q}$ and $\boldsymbol{r}$, with

(4) $V_g(v_g(\boldsymbol{q}),\boldsymbol{r}) \equiv \sum_k[v_g^k(q^k) - v_g^k(r^k)]$ and $V_{g'}(v_{g'}(\boldsymbol{q}),\boldsymbol{r}) \equiv \sum_k[v_{g'}^k(q^k) - v_{g'}^k(r^k)]$

for some regime component utility functions $v_g^k(\cdot)$ and $v_{g'}^k(\cdot)$, $k = 1, \dots, K$.

Suppose that (3) is satisfied for all $q^i = r^i$ and $\partial V_g(\cdot,\cdot)/\partial q^j \neq 0$ and so, by (3), $\partial V_{g'}(\cdot,\cdot)/\partial q^j \neq 0$. Then, adding $V(\boldsymbol{q})$ to each side of (3), partially differentiating each side with respect to q^j, partially differentiating each side with respect to q^i with $r^i = q^i$, and taking ratios, shows that the marginal rates of substitution between goods *i* and *j* (which with constant sensitivity are unaffected within each gain-loss regime by changing r^i with q^i) must be equal across regimes g and g' for all $q^i = r^i$. That contradicts the premise that the marginal rates of substitution differ across regimes in some open neighbourhood of commodity space. Thus, $\partial V_g(\cdot,\cdot)/\partial q^j \equiv \partial V_{g'}(\cdot,\cdot)/\partial q^j \equiv 0$ for any $j \neq i$, which implies that for any regimes g and g', $V_g\big(v_g(\boldsymbol{q}),\boldsymbol{r}\big)$ and

$V_{g'}(v_{g'}(\boldsymbol{q}), \boldsymbol{r})$ are additively separable in $\boldsymbol{q}$. Given that additive separability in $\boldsymbol{q}$, if $V_g(v_g(\boldsymbol{q}), \boldsymbol{r})$ and $V_{g'}(v_{g'}(\boldsymbol{q}), \boldsymbol{r})$ are not also additively separable in $\boldsymbol{r}$, and do not have gain-loss utilities whose good-by-good responses to reference points mirror their responses to bundles as in (4), then for some $\boldsymbol{q}, \boldsymbol{r}$, and *k*, changing q^k and r^k with $r^k = q^k$ induces different changes in $V_g(v_g(\boldsymbol{q}), \boldsymbol{r})$ and $V_{g'}(v_{g'}(\boldsymbol{q}), \boldsymbol{r})$, violating (3). That contradiction establishes (4), and thus (2). ■

The preferences characterized in Proposition 1 nest Farber's (2005, 2008), Kőszegi and Rabin's (2006), and Crawford and Meng's (2011) functional structure assumptions with constant sensitivity, but they are more general in two ways that are potentially important empirically: They allow the preferences over consumption bundles induced by consumption plus gain-loss utility to vary freely across regimes, with marginal rates of substitution that need not satisfy the knife-edge cross-regime restrictions implied by Kőszegi and Rabin's assumptions with constant sensitivity (see Crawford and Meng's Table 1). And they allow consumption utility, and thereby the induced preferences over bundles, *not* to be additively separable across goods.

One aspect of Proposition 1's equation (2) is important in Section 4.3's conditions for a rationalization with modelable reference points and constant sensitivity. Although with constant sensitivity a consumer's preferences and optimal choice of $\boldsymbol{q}$ for a given budget set are independent of $\boldsymbol{r}$ within a gain-loss regime, each regime's maximized values of $u(\boldsymbol{q}, \boldsymbol{r})$ vary with $\boldsymbol{r}$. (2) assigns each regime g a "loss cost" $\sum_k[-v_g^k(r^k)]$, incurred whenever any bundle $\boldsymbol{q}$ in regime g is chosen but otherwise independent of $\boldsymbol{q}$.[11] Loss costs influence a consumer's incentive to defect from an observation's bundle to some bundle in his budget set in another regime, which is potentially beneficial depending on how changing regimes alters his preferences.

[11] "Loss cost" is a slight misnomer, in that for a gain-averse consumer, if one were observed, the proper term would be "gain cost". Although (2) relates $u(\boldsymbol{q}, \boldsymbol{r})$ to the distances between each good *k*'s q_k and r_k, its $\sum_{g,k} I_g(\boldsymbol{q}, \boldsymbol{r})\mathrm{v}_\mathrm{g}^\mathrm{k}(q^k)$ term is part of the regime's preferences over $\boldsymbol{q}$, which is chosen—unlike (2)'s $\sum_{g,k} I_g(\boldsymbol{q}, \boldsymbol{r})[-\mathrm{v}_\mathrm{g}^\mathrm{k}(r^k)]$ term, which is exogenous.

3. UNMODELABLE REFERENCE POINTS

This section studies rationalization via reference-dependent preferences with unmodelable reference points. In this case we assume that a reference point can be chosen hypothetically for each observation as part of a rationalization—a nonparametric analogue of Camerer et al.'s (1997) and Farber's (2005, 2008) treatment of earnings targets as latent variables. Definition 2 defines rationalization for this case. Proposition 2 shows that the ability to hypothesize reference points gives preferences enough flexibility to rationalize any dataset, making the hypothesis of reference-dependent preferences nonparametrically irrefutable.

DEFINITION 2: [Rationalization with unmodelable reference points.] Reference-dependent preferences and an associated utility function $u(\boldsymbol{q}, \boldsymbol{r})$*, and hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$*, rationalize the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ *if and only if* $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *and* $\boldsymbol{t}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

PROPOSITION 2: [Rationalization with unmodelable reference points via preferences and an associated utility function with variable or constant sensitivity.] For any data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ *with unmodelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q}, \boldsymbol{r})$ *that are jointly continuous, increasing in* $\boldsymbol{q}$*, and decreasing in* $\boldsymbol{r}$*, and a sequence of hypothetical reference points* $\{\boldsymbol{r}_t\}_{t=1,\ldots,T}$*, that rationalize the data.*

PROOF: Recall that we denote goods by superscripts, so that $\boldsymbol{q} \equiv (q^1, \ldots, q^K)$, $\boldsymbol{q_t} \equiv (q_t^1, \ldots, q_t^K)$, and so on. Let $a^k \equiv min_{t=1,\ldots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k \geq r_t^k$; and $a^k \equiv max_{t=1,\ldots,T}\{p_t^k\} > 0$ for each *k* and *t* such that $q_t^k < r_t^k$. Define the utility function $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_k a^k q^k + \sum_k a^k (q^k - r^k)$, which is strictly increasing in $\boldsymbol{q}$, strictly decreasing in $\boldsymbol{r}$, and satisfies constant sensitivity and Proposition 1's conditions for joint continuity in that case.[12] For each observation *t*, set $\boldsymbol{r}_t = \boldsymbol{q}_t$ and consider any bundle $\boldsymbol{q} \neq \boldsymbol{q}_t = \boldsymbol{r}_t$ that (without loss of generality, given strict monotonicity in $\boldsymbol{q}$) satisfies observation *t*'s budget constraint with equality. For such bundles, $\sum_k p_t^k\,(q^k - q_t^k) = 0$ and, by the definition of the a^k,

(5) $\sum_k (a^k - p_t^k)(q^k - q_t^k) = \sum_k (a^k - p_t^k)(q^k - r_t^k) < 0$ and $\sum_k a^k (q^k - r_t^k) < 0$
and

(6) $$u(\boldsymbol{q}, \boldsymbol{r}_t) - u(\boldsymbol{q}_t, \boldsymbol{r}_t) = 2\sum_k a^k (q^k - q_t^k) = 2\sum_k a^k (q^k - r_t^k) < 0,$$

so $u(\boldsymbol{q}, \boldsymbol{r})$ rationalizes the choice of $\boldsymbol{q}_t$. This holds a fortiori for variable sensitivity. ■

The proof hypothesizes a reference point for each observation that coincides with its consumption bundle, and preferences that with those reference points put each observation's bundle at the kink of an approximately Leontief indifference curve. Reference points enter preferences additively separably and loss costs cancel out of the relevant utility comparisons. That the rationalization works entirely by varying reference points across observations shows as directly as possible that the parsimony of reference-dependent consumer theory depends on modeling or observing reference points. The proof's preferences satisfy joint continuity, constant sensitivity, and Farber's, Kőszegi and Rabin's, and Crawford and Meng's functional structure and form assumptions, so they too are nonparametrically untestable.

4. MODELABLE REFERENCE POINTS

This section studies rationalization via reference-dependent preferences with modelable or observable (henceforth "modelable") reference points. Our results for this case build on Afriat's (1967), Diewert's (1973), and Varian's (1982) results that link the existence of a rationalization via neoclassical preferences to the Generalized Axiom of Revealed Preference ("GARP"), which we restate for convenience.

4.1. GARP, Afriat's Theorem, Afriat preferences, and Afriat utility functions

DEFINITION 3: [Generalized Axiom of Revealed Preference ("GARP").] $\boldsymbol{q_s} R \boldsymbol{q_t}$ *implies* $\boldsymbol{p}_t \cdot \boldsymbol{q}_t \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_s$, *where* R

[12] In the hypothesized preferences the parameters a^k are constants, even though their estimates depend on the $\boldsymbol{q}_t$—as they would in structural econometric or other nonparametric estimation.

indicates that there is some sequence of observations $\boldsymbol{q}_h, \boldsymbol{q}_i, \boldsymbol{q}_j, \ldots, \boldsymbol{q}_t$ *such that* $\boldsymbol{p}_h \cdot \boldsymbol{q}_h \geq \boldsymbol{p}_h \cdot \boldsymbol{q}_i, \boldsymbol{p}_i \cdot \boldsymbol{q}_i \geq \boldsymbol{p}_i \cdot \boldsymbol{q}_j, \ldots, \boldsymbol{p}_s \cdot \boldsymbol{q}_s \geq \boldsymbol{p}_s \cdot \boldsymbol{q}_t$.

AFRIAT'S THEOREM: [Afriat 1967, Diewert 1973, Varian 1982.] The following statements are equivalent:

A) There exists a utility function $u(\boldsymbol{q})$ *that is continuous, non-satiated, and concave, and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$.

B) There exist numbers $\{\boldsymbol{U}_t, \lambda_t > 0\}_{t=1,\ldots,T}$ *such that*

(7) $\quad U_s \leq U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q}_s - \boldsymbol{q}_t)$ *for all* $s, t \in \{1, \ldots, T\}$

C) The data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ *satisfy GARP.*

D) There exists a non-satiated utility function $u(\boldsymbol{q})$ *that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$.

In the proof of Afriat's Theorem, which is constructive, condition B)'s inequalities (7) hold with equality. That yields a canonical set of convex rationalizing preferences and an associated utility function, which we shall call the "Afriat" preferences and utility function. The latter is piecewise linear, continuous, non-satiated, and concave. With finite data the Afriat preferences are only one of many possibilities for a rationalization (Varian 1982, Fact 4), but they play a central role in our analysis of the case with modelable reference points and constant sensitivity. Definition 4 identifies the Afriat preferences and the associated Afriat utility function.

DEFINITION 4: [Afriat preferences and associated Afriat utility function.] For data $\{\boldsymbol{p}_t, \boldsymbol{q}_t\}_{t=1,\ldots,T}$ *that satisfy GARP, or equivalently condition B) of Afriat's Theorem, the Afriat preferences are those represented by the Afriat utility function* $u(\boldsymbol{q}) = min_{t \in \{1,\ldots,T\}}\{U_t + \lambda_t \boldsymbol{p}_t \cdot (\boldsymbol{q} - \boldsymbol{q}_t)\}$, *where the* U_t *and*

λ_t are those that satisfy the binding condition B) inequalities (7) in Afriat's Theorem.

Figure 2. Afriat preferences for data that satisfy GARP

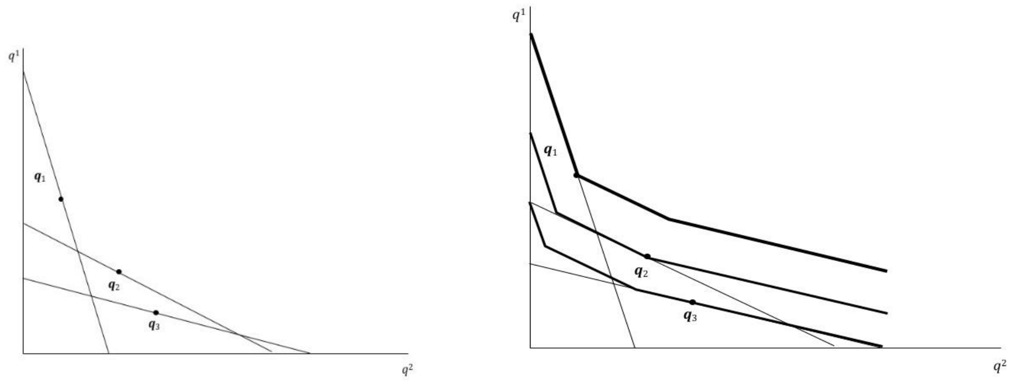


Figure 2 illustrates the Afriat preferences for a three-observation dataset. The left-hand panel shows the observations' budget sets and consumption bundles. The right-hand panel superimposes the associated Afriat indifference map, whose marginal rates of substitution are determined by the budget lines as indicated.

4.2. Variable sensitivity

We begin with a definition of rationalization via reference-dependent preferences with modelable reference points and either variable or constant sensitivity.

DEFINITION 5: [Rationalization with modelable reference points.] Reference-dependent preferences and an associated utility function $u(\boldsymbol{q},\boldsymbol{r})$ rationalize the data $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ with modelable reference points if and only if $u(\boldsymbol{q}_t,\boldsymbol{r}_t) \geq u(\boldsymbol{q},\boldsymbol{r}_t)$ for all $\boldsymbol{q}$ and $\boldsymbol{t}$ such that $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$.

Proposition 3 shows that with modelable reference points and variable sensitivity, the hypothesis of reference-dependent preferences is nonparametrically refutable only via violations of GARP within subsets of observations that share exactly the same reference point. It shows that reference-dependence adds nothing to the neoclassical model in the way of refutable implications, making this case an empirical dead end.

PROPOSITION 3: [Rationalization with modelable reference points via preferences with variable sensitivity.] For any data $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points, there exist reference-dependent preferences and an associated utility function* $u(\boldsymbol{q}, \boldsymbol{r})$ *that for each observation t and reference point* $\boldsymbol{r}_t$*, are jointly continuous and strictly increasing in* $\boldsymbol{q}$ *and that rationalize the data, if and only if every subset of the data whose observations share exactly the same reference point satisfies GARP.*

PROOF: Partition the observations into subsets $\tau^j, j = 1, \dots, J$, such that if and only if two observations $\{\boldsymbol{p}_s, \boldsymbol{q}_s, \boldsymbol{r}_s\}$ and $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$ have exactly the same reference point $\boldsymbol{r}_s = \boldsymbol{r}_t$, they are in the same subset. If there exists a reference-dependent utility function with the stated properties that rationalizes the data, then the data within any such subset must satisfy GARP within that subset, by Afriat's Theorem. Conversely, suppose the data within each such subset satisfies GARP. Let $b^k \equiv min_{t=1,\dots,T}\{p_t^k\}$, so that $0 < b^k \leq p_t^k$, and let $\boldsymbol{b} \equiv (b^1, \dots, b^K)$. For any subset $\tau^j$and observation $t \in \tau^j$, let the indicator function $I_{\tau^j}(t) = 1$ if the observation $t \in \tau^j$ and $I_{\tau^j}(t) = 0$ otherwise, and let $u(\boldsymbol{q}, \boldsymbol{r}) \equiv \sum_j I_{\tau^j}(t) U^j(\boldsymbol{q}, \boldsymbol{r}_t)$, where $U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$, Definition 4's Afriat utility function for observations in τ^j, with the U_ρ^j and λ_ρ^j taken from τ^j's binding condition B) inequalities (6) in Afriat's Theorem, and with $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ augmented by a common, additively separable loss cost. If τ^j is a singleton subset the terms in $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ other than the loss cost follow observation *t*'s budget line in commodity space. If not those terms follow the minimum of τ^j's observations' budget lines, as in the right-hand panel of Figure 2. Either way, $\boldsymbol{r}_t$ completely determines the $\boldsymbol{p}_\rho$ and $\boldsymbol{q}_\rho$ for all $\rho \in \tau^j$, as is required to determine $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$. For each $\boldsymbol{r}_t$, $u(\boldsymbol{q}, \boldsymbol{r}_t)$ and $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ are continuous and increasing in $\boldsymbol{q}$. For any subset $\tau^j$and observation $t \in \tau^j$ and any $\boldsymbol{q}$ with $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$, using τ^j's binding condition B) inequalities (6) for the preferences in that subset,

(8) $$U^j(\boldsymbol{q}, \boldsymbol{r}_t) \equiv min_{\rho \in \tau^j}\{U_\rho^j + \lambda_\rho^j \boldsymbol{p}_\rho \cdot (\boldsymbol{q} - \boldsymbol{q}_\rho)\} - \boldsymbol{b} \cdot \boldsymbol{r}_t$$

$$\leq U_t^j + \lambda_t^j \boldsymbol{p}_t \cdot (\boldsymbol{q}_t - \boldsymbol{q}_t) - \boldsymbol{b} \cdot \boldsymbol{r}_t = U_t^j - \boldsymbol{b} \cdot \boldsymbol{r}_t \equiv U^j(\boldsymbol{q}_t, \boldsymbol{r}_t). \blacksquare$$

The proof's restrictions on $\boldsymbol{b}$ make $U^j(\boldsymbol{q}, \boldsymbol{r}_t)$ for any given $\boldsymbol{r}_t$ consistent with additively separable consumption and gain-loss utilities, strictly increasing in $\boldsymbol{q}$. Unlike Proposition 2, Proposition 3 does *not* claim that $u(\boldsymbol{q}, \boldsymbol{r})$ is jointly continuous in $\boldsymbol{q}$ and $\boldsymbol{r}$ or continuous or decreasing in $\boldsymbol{r}$. For, a rationalization might require preferences that violate joint continuity because observations with nearby $\boldsymbol{r}_t$ values have very different budget sets. We have not tried to characterize rationalizability via a jointly continuous $u(\boldsymbol{q}, \boldsymbol{r})$ in this case, and we suspect that informative results on that are unavailable. If, however, the data are generated by preferences that satisfy joint continuity, Proposition 3's rationalizations should converge to a jointly continuous limiting $u(\boldsymbol{q}, \boldsymbol{r})$ as the data become rich in the sense of Definition 6.

4.3. Constant sensitivity

We turn to the case of modelable reference points and constant sensitivity, as Kőszegi and Rabin (2006) sometimes and Crawford and Meng (2011) always assume. In this case reference-dependent preferences can rationalize some data that violate GARP in the aggregate. The model of reference points can be used to sort the data objectively into gain-loss regimes. Within each regime, preferences over consumption bundles must be constant. Thus GARP for each regime's observations is necessary for a rationalization. By Afriat's Theorem it ensures that there are regime preferences that preclude beneficial defections from any observation's consumption bundle in the regime to any other bundle in that regime in the observation's budget set. In addition, a rationalization requires there is a choice of rationalizing regime preferences that preclude beneficial defections from any observation's bundle to any bundle in another regime in the observation's budget set, taking Proposition 1's loss costs into account. Together these two sets of conditions are necessary and sufficient for a reference-dependent rationalization. Proposition 4 records them as a benchmark.

Recall that g is a binary vector of length K, with kth component 1 if $q_t^k \geq r_t^k$ and 0 if $q_t^k < r_t^k$. Let $G(g; \boldsymbol{r})$ be the set of consumption bundles $\boldsymbol{q}$ that are in regime g for reference point $\boldsymbol{r}$**.** Let $H(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g) \equiv \{t \in \{1, \dots, T\} \mid \boldsymbol{q}_t \in G(g; \boldsymbol{r}_t)\}$ be the set of indices of observations $(\boldsymbol{q}_t, \boldsymbol{r}_t)$ for which $\boldsymbol{q}_t$ is in regime g for $\boldsymbol{r}_t$.

PROPOSITION 4: [Rationalization with modelable reference points via preferences with constant sensitivity.] Suppose that reference-dependent preferences are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences, satisfy constant sensitivity and are jointly continuous, and that they satisfy Proposition 1's equation (2). Consider data $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. Then the statements A) and B) are equivalent:*

A) There exists a jointly continuous reference-dependent utility function $u(\boldsymbol{q}, \boldsymbol{r})$ *that satisfies constant sensitivity; is strictly increasing in* $\boldsymbol{q}$ *and strictly decreasing in* $\boldsymbol{r}$*; and that rationalizes the data* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$.

B) Each regime's data satisfy GARP within the regime; and there is some combination of rationalizing regime preferences over bundles, with continuous, strictly increasing consumption utility function $V(\cdot)$ *and component utility functions* $v_g^k(\cdot)$*, such that for each observation* $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *with* $t \in H(\{\boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}; g)$ *and each* $\boldsymbol{q} \in G(g'; \boldsymbol{r})$ *with* $g' \neq g$ *and* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$,

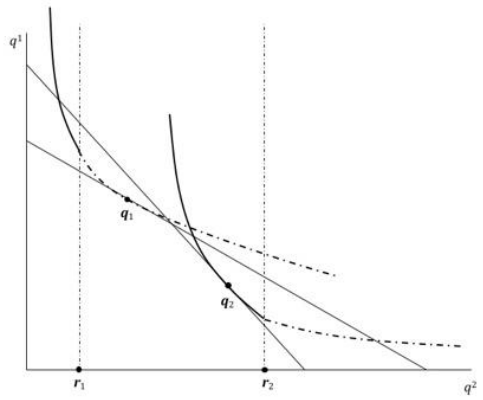
$$(9) \quad u(\boldsymbol{q}, \boldsymbol{r}_t) \equiv V(\boldsymbol{q}) + \textstyle\sum_k [v_{g'}^k(q^k) - v_{g'}^k(r_t^k)] \leq V(\boldsymbol{q}_t) + \textstyle\sum_k [v_g^k(q_t^k) - v_g^k(r_t^k)] \equiv u(\boldsymbol{q}_t, \boldsymbol{r}_t).$$

Condition (9) precludes beneficial defections from any observation's consumption bundle within as well as across gain-loss regimes. However, although GARP is necessary and sufficient to preclude within-regime defections, there appears to be no comparably simple condition to preclude cross-regime defections.[13]

[13] Online Appendix A defines a notion of loss aversion that generalizes Tversky and Kahneman's (1991) definition assuming constant sensitivity to the multi-good case and relaxes their assumption of additive separability across goods. A Corollary of Proposition 4 then shows that GARP within each gain-loss regime and the existence of a set of rationalizing regime preferences that satisfy a condition that is weaker than loss aversion is also sufficient for a rationalization. Loss aversion is usually viewed

Figures 3 illustrates Proposition 4's conditions. There, the entire dataset violates GARP, with observation 1's consumption bundle chosen in 1's budget set over observation 2's bundle, and vice versa. However, the observations' reference points put their bundles in different regimes (good-2 gain for observation 1, good-2 loss for observation 2), so constant sensitivity allows them to have different regime preferences. Each regime's single observation trivially satisfies GARP for a within-regime rationalization; and condition (9) can be shown to be satisfied.

Figure 3. Rationalizing data that violate GARP via reference-dependent preferences with constant sensitivity (solid lines for the loss map, dashed lines for the gain map)



Although Proposition 4's conditions are necessary and sufficient for a rationalization, they are not directly applicable because with finite data there is a range of preferences that rationalize a gain-loss regime's data (Varian 1982, Fact 4) and condition B) rests on an unspecified choice among those rationalizing regime preferences. Finding a choice that precludes beneficial cross-regime defections involves complex trade-offs, because preferences that reduce the gain from defecting *from* bundles in a regime increase the gain from defecting *to* bundles in the regime.

Proposition 5 uses Proposition 4's conditions to derive directly applicable sufficient conditions by specializing the choice of rationalizing regime preferences to Definition 4's Afriat regime preferences. Proposition 5's sufficient conditions are not necessary; but Proposition 6 shows that in the limit as the data become rich

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as an empirically well-supported assumption with important behavioral implications, but not as one that is linked to the *existence* of a reference-dependent rationalization, as it is in the Corollary.

(Definition 6), so that each regime's range of convexified rationalizing regime preferences collapses on its Afriat preferences (Mas-Colell 1978 and Forges and Minelli 2009), Proposition 5's sufficient conditions are asymptotically necessary.

As before, let $G(g;\boldsymbol{r})$ be the set of consumption bundles $\boldsymbol{q}$ that are in regime g for reference point $\boldsymbol{r}$**.** Let $H\big(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g\big) \equiv \{t \in \{1,\dots,T\} \,\big|\, \boldsymbol{q}_t \in G(g;\boldsymbol{r}_t)\}$ be the set of indices of observations $(\boldsymbol{q}_t,\boldsymbol{r}_t)$ for which $\boldsymbol{q}_t$ is in regime g for $\boldsymbol{r}_t$. Write the Afriat rationalizing utility function for gain-loss regime g,

$$\text{(10)}\qquad \begin{aligned} u^g(\boldsymbol{q},\boldsymbol{r}) - V(\boldsymbol{r}) &\equiv V(\boldsymbol{q}) + \textstyle\sum_k v_g^k(q^k) - \sum_k v_g^k(r^k) - V(\boldsymbol{r}) \\ &\equiv min_{\rho\in H(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho^g + \lambda_\rho^g \boldsymbol{p}_\rho \cdot (\boldsymbol{q}-\boldsymbol{q}_\rho)\} \\ &\quad -min_{\rho\in H(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho^g + \lambda_\rho^g \boldsymbol{p}_\rho \cdot (\boldsymbol{r}-\boldsymbol{q}_\rho)\}, \end{aligned}$$

with the U_ρ^g and λ_ρ^g from regime g's binding B) inequalities (7) from Afriat's Theorem applied to regime g.

> *PROPOSITION 5: [Rationalization with modelable reference points via preferences with constant sensitivity, using the Afriat regime preferences.] Suppose that reference-dependent preferences are defined over K ≥ 2 goods, that reference-dependence is active for all K goods, that the preferences, satisfy constant sensitivity and are jointly continuous, and that they satisfy Proposition 1's equation (2). Consider data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$ *with modelable reference points. Then if (i) each regime's data satisfy GARP within the regime; and (ii) if for the Afriat regime preferences and any observation* $t \in H\big(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g\big)$ *with* $\boldsymbol{q}_t \in G(g;\boldsymbol{r}_t)$ *and a bundle* $\boldsymbol{q} \in G(g';\boldsymbol{r_t})$ *with* $g' \neq g$ and $\boldsymbol{p_t}\cdot\boldsymbol{q} \leq \boldsymbol{p_t}\cdot\boldsymbol{q_t}$,

$$\text{(11)}\qquad \begin{aligned} u(\boldsymbol{q},\boldsymbol{r}_t) - \mathrm{V}(\boldsymbol{r}_t) &\equiv \min_{\rho\in H(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho^{g'} + \lambda_\rho^{g'} \boldsymbol{p}_\rho \cdot (\boldsymbol{q}-\boldsymbol{q}_\rho)\} \\ &\quad -\min_{\rho\in H(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g')}\{U_\rho^{g'} + \lambda_\rho^{g'} \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_t-\boldsymbol{q}_\rho)\} \\ &\leq min_{\rho\in H(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho^g + \lambda_\rho^g \boldsymbol{p}_\rho \cdot (\boldsymbol{q}_t-\boldsymbol{q}_\rho)\} \\ &\quad -min_{\rho\in H(\{\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T};g)}\{U_\rho^g + \lambda_\rho^g \boldsymbol{p}_\rho \cdot (\boldsymbol{r}_t-\boldsymbol{q}_\rho)\} \equiv u(\boldsymbol{q}_t,\boldsymbol{r}_t) - V(\boldsymbol{r}_t). \end{aligned}$$

with the U_ρ^g, $U_\rho^{g'}$, λ_ρ^g, and $\lambda_\rho^{g'}$ *from regimes* g's and g''s *binding condition B) inequalities (6) from Afriat's* Theorem *applied within those regimes, then there exists a jointly continuous reference-dependent utility function* $u(\boldsymbol{q},\boldsymbol{r})$ *that satisfies constant sensitivity, is strictly increasing in* $\boldsymbol{q}$ *and strictly decreasing in* $\boldsymbol{r}$, *and that rationalizes the data* $\{\boldsymbol{p}_t,\boldsymbol{q}_t,\boldsymbol{r}_t\}_{t=1,\dots,T}$.

PROOF: Condition (i) ensures that the utility function $u(\boldsymbol{q},\boldsymbol{r}) \equiv V(\boldsymbol{q}) + \sum_{g,k} I_g(\boldsymbol{q},\boldsymbol{r})\, u^g(\boldsymbol{q},\boldsymbol{r})$, with the $u^g(\boldsymbol{q},\boldsymbol{r})$ based on the Afriat regime preferences as in (9), is jointly continuous, strictly increasing in $\boldsymbol{q}$, and strictly decreasing in $\boldsymbol{r}$; and rationalizes each regime g's data within the regime. Combining (10) with Proposition 4's condition (9) then yields condition (ii)'s inequality (11). ■

Proposition 5 relies essentially on our (and Farber's 2005, 2008, Kőszegi and Rabin's 2006, and Crawford and Meng's 2011) assumption that consumption and gain-loss utility enter preferences additively separably. It also relies essentially on the choice of the Afriat regime preferences.[14] Operationalizing (11)'s conditions precluding beneficial defections across gain-loss regimes requires linking (2)'s loss costs to things that can be estimated from the data, not only at particular points but as functions of $\boldsymbol{r}$. Proposition 5 uses Afriat regime preferences to estimate each regime's loss costs and induced preferences over consumption bundles simultaneously. That is possible because Proposition 1 and our assumptions identify the cross-regime differences in loss costs, which are all that matter for a rationalization.

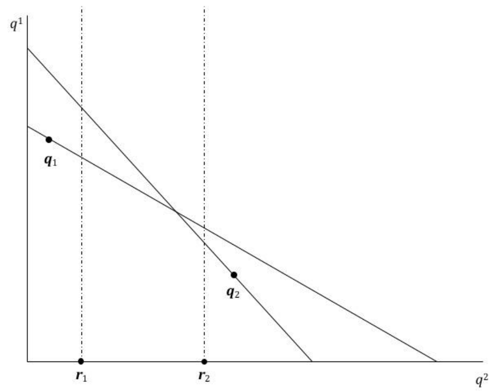
Importantly, although joint continuity and constant sensitivity require gain-loss utility $\sum_k[v_g^k(q^k) - v_g^k(r_t^k)]$ to be additively separable across goods by Proposition 1, Proposition 5 does *not* require consumption utility $V(\boldsymbol{q})$ to be additively separable across goods. This allows us to relax Farber's, Kőszegi and Rabin's, and Crawford

[14] Varian's (1982, Fact 4) bounds for the neoclassical case don't imply that all rationalizing preferences (here, rationalizing regime preferences) are convex, but examples show that requiring such convexity involves a loss of generality for the rationalizing regime preferences in Proposition 4. Proposition 5 avoids that difficulty by using the Afriat regime preferences, which are convex by construction.

and Meng's assumption that the sum of consumption and gain-loss utility that determines consumer demand is additively separable across goods.

Figure 4 illustrates Propositions 4 and 5's conditions. There, as in Figure 3, the entire dataset violates GARP; the observations' reference points put their bundles in different gain-loss regimes; and each regime's single observation trivially satisfies GARP within its regime. But now there is no choice of rationalizing regime preferences that is consistent with a rationalization. That would require regime preferences that continuously connect a loss-regime indifference curve through observation 1's bundle to a gain-regime indifference curve that would cut into observation 2's budget set and stay outside of observation 1's budget set, thus passing northeast of 2's bundle; and also loss- and gain-regime indifference curves satisfying the analogous conditions interchanging observations 1 and 2. Such indifference curves are inconsistent with the optimality of each observation's bundle in its budget set. (Loss costs might change the calculations, but they cannot help to rationalize both observations at once because they favor one observation's bundle at the other's expense.) This argument is general in that, like Proposition 4, it does not rely on the choice of rationalizing regime preferences. Online Appendix B uses Figure 4's example to illustrate the use of Proposition 5's condition (11) to show that the Afriat regime preferences, in particular, are inconsistent with a rationalization.

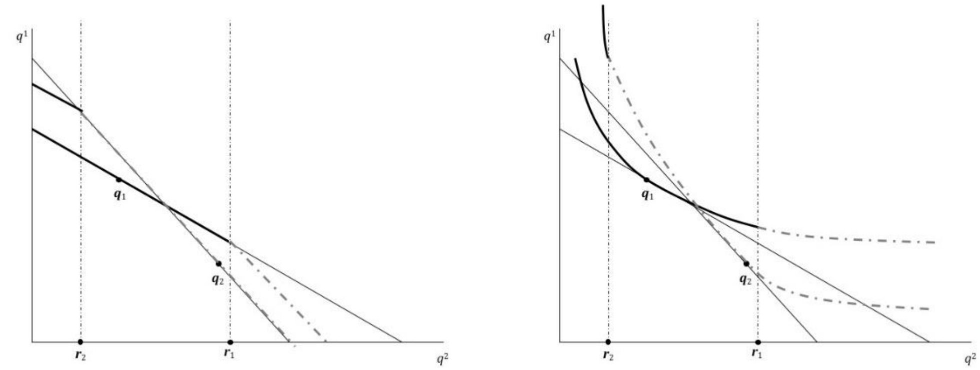
Figure 4. Failing to rationalize data that violate GARP via reference-dependent preferences with constant sensitivity



4.4. Asymptotic necessity of Proposition 5's sufficient conditions with rich data

As already noted, Proposition 5's sufficient conditions fall short of necessity because they rely on a particular choice of the Afriat rationalizing regime preferences. With finite data that choice is not without loss of generality. In Figure 5 the entire dataset violates GARP; the Afriat regime preferences in the left-hand panel violate loss aversion and fail to yield a rationalization; but the convex, non-Afriat rationalizing regime preferences in the right-hand panel do yield a rationalization.

Figure 5. A rationalization may require non-Afriat rationalizing regime preferences (solid lines for the loss map, dashed lines for the gain map)



Although empirical revealed-preference analysis is undoubtedly a finite-data enterprise, studying the limit as the data become rich in the sense of Definition 6 reveals what happens to the conditions for a rationalization in large datasets. Proposition 6 applies Mas-Colell's (1978) and Forges and Minelli's (2009) approximation results for (or including) neoclassical consumer theory, showing that as the data become rich, the ranges of convexified rationalizing preferences collapse on Afriat's rationalizing preferences, thereby eliminating Varian's (1982, Fact 4) multiplicity of convexified rationalizing preferences. Applying Mas-Colell's and Forges and Minelli's results gain-loss regime by regime shows that in the limit, if the Afriat regime preferences do not yield a rationalization, neither can any other combination of rationalizing regime preferences. Put another way, Proposition 5's sufficient conditions for a rationalization are asymptotically necessary.[15] Proposition

[15] Like Definition 6, Proposition 6 does *not* restrict consumption observations beyond requiring that each regime's data always satisfy GARP. Forges and Minelli's (2009) analysis of neoclassical preferences with nonlinear, nonconvex budget sets may appear to be dual to our analysis of possibly

6's use of the Afriat regime preferences also ensures that if the true preferences are jointly continuous and monotonic, so will be the limiting estimated $u(\boldsymbol{q}, \boldsymbol{r})$.

Following Forges and Minelli (2009), consider a sequence of observations $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T} \in \mathbb{R}_+^{\boldsymbol{2K+1}}$, with $T \to \infty$. For nonsatiated preferences, $(\boldsymbol{p}_t, \boldsymbol{q}_t)$ uniquely defines a linear budget set B_t, and a dataset is a finite collection $\{B_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ where $\boldsymbol{q_t} \in B_t \subset \mathbb{R}_{++}^{\boldsymbol{K}}$ and $\boldsymbol{r_t} \in \mathbb{R}_{++}^{\boldsymbol{K}}$. Let $\mathbb{B}$ be the set of all nonempty and compact linear budget sets $B \subset \mathbb{R}_{++}^{K}$. Consider a sequence of unions of {reference point × budget set} combinations that become richer and richer, eventually tending to the entire set $\mathbb{B} \times \mathbb{R}_{++}^{\boldsymbol{K}}$.

> DEFINITION 6: [Rich data with observable reference points.] Let $\boldsymbol{K}(\boldsymbol{X})$ be the set of all nonempty compact subsets of X. Endowed with the Hausdorff metric, $\boldsymbol{K}(\boldsymbol{X})$ is a separable metric space. $\mathbb{B} \times \mathbb{R}_{++}^{\boldsymbol{K}} \subset \boldsymbol{K}(\boldsymbol{X})$ inherits those properties. Let $\boldsymbol{C_T}$ be a collection of T elements $\{B_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ of $\mathbb{B} \times \mathbb{R}_{++}^{\boldsymbol{K}}$, and consider an increasing sequence of collections $\boldsymbol{C_1} \subset \boldsymbol{C_2} \subset \cdots \subset \boldsymbol{C_{T-1}} \subset \boldsymbol{C_T} \dots$ The data are rich if and only if the closure of their union is dense in $\mathbb{B} \times \mathbb{R}_{++}^{\boldsymbol{K}}$: $\overline{\bigcup_T \boldsymbol{C_T}} = \mathbb{B} \times \mathbb{R}_{++}^{\boldsymbol{K}}$.

Definition 6 requires richness of data only for {reference point × budget set} combinations, *not* for consumption bundles, which would implicitly rule out nonconvex preferences. Such richness means that as $T \to \infty$, the data come to include {reference point × budget set} combinations as close as desired to any possible combination. Mas-Colell's and Forges and Minelli's approximation results show that for consumer theory with competitive budget sets, if the consumer's demand system satisfies an income-Lipschitz condition,[16] then in the limit rich data uniquely determine each regime's *convexified* rationalizing indifference map, making it effectively observable so it can serve as a complete summary of the regime's data.

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nonconvex, reference-dependent preferences with linear budget sets. However, unlike in their analysis, a reference-dependent consumer can change her/his preferences by changing regimes.

[16] See Uzawa (1960). If demands are normal, for example, then they are income-Lipschitz.

PROPOSITION 6: [Sufficient and asymptotically necessary conditions for a rationalization with observable reference points and rich data, via preferences with constant sensitivity.] Consider data $\{B_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$, *and assume observable reference points, rich data, and constant sensitivity. For each* T *and each* t, *sort the data into gain-loss regimes, assume that each regime's data satisfy GARP, and use the Afriat regime preferences to obtain a candidate global preference ordering, represented by the utility function* $u(\boldsymbol{q}, \boldsymbol{r})$. *If* $u(\boldsymbol{q}_t, \boldsymbol{r}_t) \geq u(\boldsymbol{q}, \boldsymbol{r}_t)$ *for all* $\boldsymbol{q}$ *such that* $\boldsymbol{p}_t \cdot \boldsymbol{q} \leq \boldsymbol{p}_t \cdot \boldsymbol{q}_t$ *is violated for infinitely many T, then there does not exist, for* any *choice of rationalizing regime preferences, a utility function that rationalizes the data.*

PROOF: GARP within each regime is plainly necessary for a rationalization. Suppose that it is satisfied, but that the second condition is violated somewhere for infinitely many *T*. Any such violation must be strict, occurring throughout an open neighborhood in the space of possible {budget set × reference point} combinations $\mathbb{B} \times \mathbb{R}^{\boldsymbol{K}}_{++}$. That space is finite-dimensional, Euclidean, and can be uniformly bounded without loss of generality. By the Bolzano–Weierstrass Theorem, the sequence of {budget set × reference point} combinations that yield violations of the second condition must have a convergent subsequence, throughout which the violations remain in a constant neighborhood of $\mathbb{B} \times \mathbb{R}^{\boldsymbol{K}}_{++}$. For each collection $\boldsymbol{C_T}$ of *T* elements $\{B_t, \boldsymbol{r}_t\}_{t=1,\dots,T}$ *of* $\mathbb{B} \times \mathbb{R}^{\boldsymbol{K}}_{++}$ and regime *i*, let $\mathcal{R}^i_T$ be the set of upper semicontinuous, monotonic preference relations that rationalize regime *i*'s data. By Forges and Minelli (2009, Proposition 5), if regime *i*'s choice correspondence h^i is monotonic and satisfies GARP, then $\mathcal{R}^i(h^i)$ is the unique preference relation that generates h^i. Consider an increasing sequence of collections $\boldsymbol{C_1} \subset \boldsymbol{C_2} \subset \cdots \subset \boldsymbol{C_{T-1}} \subset \boldsymbol{C_T} \dots$ For each regime *i*, we obtain a decreasing sequence $\dots\ \mathcal{R}^i_T \supset \mathcal{R}^i_{T-1}$, ... By Forges and Minelli (2009, Proposition 6), that sequence is well defined and in the limit coincides with the unique preference relation that generates the limiting choice correspondence $h^i\colon \mathbb{B} \to \mathbb{R}^{\boldsymbol{K}}_{++}$: If regime *i*'s limiting choice correspondence $h^i\colon \mathbb{B} \to \mathbb{R}^{\boldsymbol{K}}_{++}$ has closed values, is monotonic and upper hemicontinuous, and satisfies GARP, then $\bigcap_T \mathcal{R}_T^i = \{\mathcal{R}^i(h^i)\}$. By Hausdorff continuity, any selection of rationalizing regime maps $\mathcal{R}_T^i$ will also violate the second condition in that neighborhood for infinitely many T, contradicting the optimality of the associated consumption bundles. ■

5. EMPIRICAL ILLUSTRATION: FARBER'S (2005, 2008) AND CRAWFORD AND MENG'S (2011) CABDRIVERS REVISITED

Proposition 5's finite-data sufficient conditions for a reference-dependent rationalization in the empirically useful case of constant sensitivity and modelable reference points suggest a simple nonparametric estimation procedure: (i) check GARP gain-loss regime by regime; (ii) assuming GARP is satisfied in each regime, calculate each regime's Afriat rationalizing regime utilities; and (iii) use them to check Proposition 5's no-defection condition (11), observation by observation. This procedure inherits most of the simplicity and computational tractability of Diewert's (1973) and Varian's (1982) linear-programming methods for the neoclassical case. In this section we use it to reconsider Farber's (2005, 2008) and Crawford and Meng's (2011) structural econometric analyses of cabdrivers' labor supply.

Our empirical illustration follows Farber's and Crawford and Meng's analyses, with the following exceptions: We deviate from their assumption that drivers have homogeneous preferences, as usually assumed in the labor literature, instead allowing unrestricted heterogeneity of drivers' preferences, as usually assumed in nonparametric demand analyses. Although our theoretical analysis applies equally with driver homogeneity, drivers' choice behavior is sufficiently varied to make allowing heterogeneity important. Further, in the hope of guiding future specification choices, we compare several expectations-based and recent experience-based alternatives to Crawford and Meng's sample proxies for rational-expectations reference points. Finally, like Crawford and Meng but unlike Farber, who considered only earnings targets, we allow the form of reference-dependence to be either in earnings alone, in hours alone, or in both earnings and hours.

In this section's two-good illustration, GARP (Definition 3) reduces to the Weak Axiom of Revealed Preference ("WARP"), $\boldsymbol{q_s} R \boldsymbol{q_t}$ *and* $\boldsymbol{q}_s \neq \boldsymbol{q}_t$ *implies* not $\boldsymbol{q_t} R \boldsymbol{q_s}$, with R defined as for GARP. WARP is then necessary and sufficient for a neoclassical rationalization. We now use the term WARP for both conditions.

5.1. Data

Like Crawford and Meng (2011) we use Farber's (2005, 2008) original dataset.[17] Farber collected 593 "trip sheets" for 13461 trips by 21 drivers between June 1999 and May 2001. Some of these trip sheets refer to the same day for the same driver so so there are data on 584 shifts. (In addition to the 15 drivers whose choices Farber and Crawford and Meng analyzed, we consider 6 with small samples of 10 or fewer shifts, which they excluded, because our methods make some allowance for sample size.) Each trip sheet records the driver's name, hack number, date, each fare's start time and location, each fare's end time and location, and the fare paid.

Table 1: Descriptive statistics, driver by driver

	T	Working Hours	Driving Hours	Waiting Hours	Break Hours	Earnings ($)	Wage ($/hr)	Afriat Efficiency
river 1	39	6.85	4.32	2.53	0.90	157.58	36.41	0.9926
Driver 2	14	3.89	2.78	1.11	2.41	97.10	34.68	1
Driver 3	6	6.66	4.61	2.05	0.74	163.42	36.19	1
Driver 4	40	6.28	4.52	1.76	0.39	147.51	33.02	0.9978
Driver 5	23	6.46	3.98	2.48	2.11	144.96	38.12	0.9961
Driver 6	6	8.62	6.48	2.14	2.42	205.00	33.49	1
Driver 7	24	6.47	4.42	2.05	0.74	160.71	36.69	0.9985
Driver 8	37	7.78	5.13	2.64	0.86	172.44	34.23	0.9919
Driver 9	19	7.17	5.47	1.70	0.54	162.02	30.61	0.9995
Driver 10	45	6.35	3.90	2.45	1.65	133.19	33.83	0.9935
Driver 11	6	7.15	5.22	1.93	0.71	182.81	35.50	1
Driver 12	13	6.15	4.03	2.13	0.55	157.95	39.44	0.9972
Driver 13	10	7.03	4.72	2.31	0.53	154.19	33.26	1
Driver 14	17	7.06	4.49	2.57	0.64	165.84	37.37	0.993
Driver 15	8	10.82	7.64	3.17	0.19	228.26	29.92	1
Driver 16	70	6.84	4.56	2.28	0.93	172.01	37.72	0.9937
Driver 17	10	5.88	3.71	2.17	0.54	144.57	39.10	0.9946
Driver 18	72	8.53	5.84	2.69	0.60	203.05	35.07	0.9845
Driver 19	33	6.91	4.63	2.29	0.97	163.51	36.01	0.9884
Driver 20	46	7.10	4.80	2.30	0.67	156.23	32.73	0.9859
Driver 21	46	5.32	3.66	1.66	0.24	128.97	35.62	0.9917

Table 1 reports descriptive statistics driver by driver. The values are the same as those in Farber's (2005) Table B1, except for the hourly wage variable in the final

[17] The data are posted with Farber (2008) at https://www.aeaweb.org/aer/data/june08/20030605_data.zip, and with Crawford and Meng (2011) at https://www.aeaweb.org/aer/data/aug2011/20080780_data.zip. They are adequate for an illustration but smaller than ideal for a nonparametric analysis, especially allowing unrestricted driver heterogeneity.

column. Farber's wage variable is income per hour spent working, where working time is defined to be the sum of time spent driving with a fare-paying passenger aboard and time spent waiting for the next passenger (and not earning anything). By contrast, our wage variable is earnings per hour spent driving: We treat waiting time as a fixed cost, which we assume varies exogenously from shift to shift, with weather, the number of customers, and so on. Because waiting time does not respond directly to earnings, treating it as a fixed cost seems better suited to the model's logic.

This difference in the definition of the wage variable has important consequences in our nonparametric analysis. Table 1 shows that waiting times range from about a quarter to 40 percent of Farber's drivers' time on a shift. Including waiting in working time, as in Farber's and Crawford and Meng's analyses, would make shift-to-shift wage variation cause each observation's budget constraint to pivot around its zero-earnings end. As a result, a driver's budget constraints would never cross, he would trivially satisfy WARP, and a nonparametric analysis would provide nothing more than a meaningless recapitulation of his data. By contrast, treating waiting times as a fixed cost makes it possible for a driver's budget constraints to cross, so that WARP is a meaningful restriction and a nonparametric analysis can, to the extent that WARP is sastisfied, provide a meaningful interpretation of the data.[18]

Figure C.1 in Online Appendix C shows the entire dataset graphically, with each driver's budget constraints and choices. Our use of waiting times as a fixed cost makes a driver's budget constraints cross frequently. Mass points in consumption bundles, often taken to be a tell for reference-dependence, are rare. Here their rarity is merely a symptom of the modeled variation of reference points across observations.

5.2 Alternative models

We consider alternative models that vary in three dimensions: how reference points are modeled, the form of reference-dependence, and whether additive separability across goods is imposed. Regarding how reference points are modelled, we consider

[18] Table 1's final column reports each driver's Afriat efficiency index. An index of 1 means that the driver satisfies WARP; an index less than 1 means that he does not satisfy WARP. 6 of Farber's 21 drivers satisfy WARP; the remaining 15 fail WARP. Except for driver 2, the drivers who satisfy WARP are those that Farber (2005, 2008) and Crawford and Meng (2011) excluded due to small sample size. For those drivers, the data may simply be under-powered to reject the neoclassical model. We return to the issue of correcting for power to reject in some detail in Section 5.3.

two classes of models: one that treats them as proxied rational expectations as in Crawford and Meng (2011) and one that treats them as determined by recent experience. Expectations-based reference points are based on “leave-one-out” means: sample averages of the driver’s choices, excluding the current shift to avoid confounding as in Crawford and Meng. Experience-based reference points are based on one-shift lags. In each case we consider both unconditional versions of the model and versions that condition on variables that change demand and thus influence waiting time, as in Farber (2005, 2008) and Crawford and Meng (2011): weather (rain, snow, or dry) and/or time of day (day or night). For example, if it is raining in the current shift and we are considering a recent experience-based model, we take the driver’s reference point to be his realized earnings and/or hours on his last wet shift. This gives us, in all, 18 alternative models of reference points.

Regarding the form of reference-dependence, we compare the neoclassical model with models in which reference-dependence is with respect to hours only, earnings only, or both hours and earnings.[19]

Regarding additive separability across goods, recall that Proposition 1 allows us to relax Kőszegi and Rabin’s (2006), Farber’s (2005, 2008), and Crawford and Meng’s (2011) assumption that preferences are additively separable across goods, which is an empirically important aspect of our generalization of their specifications. We compare neoclassical and reference-dependent models that do or do not impose additive separability across goods. We maintain throughout Proposition 1’s (and Kőszegi and Rabin’s, Farber’s, and Crawford and Meng’s) assumption that preferences are additively separable across consumption utility and gain-loss utility.[20]

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[19] We condition on models assuming different forms of reference-dependence, rather than estimating the form or reference-dependence directly. The latter would be computationally complex because the Afriat rationalizing regime preferences are not invariant to merging regimes.

[20] Debreu’s (1960) necessary and sufficient “double cancellation” condition shows that with two goods, the Afriat rationalizing regime preferences preclude additive separability in regimes with more than one observation. With additive separability we therefore use Varian’s (1983, Theorem 6) linear program, specializing inequalities like those in condition B) of Afriat’s Theorem, in place of WARP; and we use a version of condition (11) modified to require additive sepaability across goods. For the pass rate and Selten measures, we design and implement a search algorithm using the fact that that the pass rate for the additively separable model cannot exceed that for the non-additively separable model, to make it more computationally efficient. Details and code are contained in our Replication files.

5.3. Model comparisons: Pass rates and Selten's measure of predictive success

A natural starting point for assessing the success of alternative models is the simplest possible measure of agreement between theory and data, the pass rate, known in this context as the Houtman-Maks index (Houtman and Maks 1985). The pass rate for a given model, for driver *i*, denoted $r^i \in [0,1]$, is defined as the maximal proportion of the data that is consistent with the model.

However, pass rates are not an adequate criterion for assessing models of varying flexibility, for two reasons. First, the empirical content of a model depends on the setting in which behavior is observed. Neoclassical consumer theory, for instance, can be highly restrictive or entirely without content depending on the number of observations and how often budget lines cross: The fewer the number of observations the harder it is to detect violations; and even with many data points, if the budget lines never cross, the neoclassical model can never be rejected. Second, for a given setting, more flexible models must fit better, or at least no worse. Reference-dependent models of consumer behavior are more flexible than neoclassical models, and this accounts for much of the profession's skepticism about their parsimony.

To control for flexibility we use the leading nonparametric measure of predictive success, due to Selten and Krischker (1983) and Selten (1991) (see also Beatty and Crawford 2011). The Selten measure, as we call it, levels the playing field between more- and less-flexible models in a well-defined, objective way, similar in spirit to the adjusted R^2 or the Akaike Information Criteria in structural econometrics, whereby model fit and likelihood are penalized for the model's number of free parameters.

In our current illustration the Selten measure is defined as follows, building on Beatty and Crawford's application to neoclassical consumer theory. First, using the data separately for each driver *i* and reference-point model, we calculate what Selten calls the "area" of each model being compared, $a^i \in [0,1]$. The area is the size of the set of all possible model-consistent choice sets relative to the size of the set of all feasible choice sets, or equivalently the probability that uniformaly randomly generated data are consistent with the model. Areas close to zero indicate that the driver's choices are highly constrained by the theory. Areas close to one indicate that the model can rationalize almost any choices. Thus the area reflects all three considerations mentioned above: the number of observations, the extent to which budget lines cross, and the model's flexibility.[21] We then use the area to penalize the pass rate. Selten (1991) notes that successful theories combine small values of a^i with large values of r^i, and gives an axiomatic argument for a measure of predictive success given by the difference $m(r^i, a^i) \equiv r^i - a^i$.[22] The resulting measure $m \in [-1,1]$ can be viewed as a pass rate corrected for the theory's power to detect rejections. As $m \to 1$, the theoretical restrictions become tighter and yet behavior satisfies them: a highly successful model. As $m \to -1$, the restrictions become vacuous and yet behavior fails to satisfy them: a pathologically bad model. As $m \to 0$, the restrictions approach random compliance: a harmless but useless model.

5.4 Empirical Procedure

We estimate driver-by-driver and model by model. For a given driver and the neoclassical model we fix whether preferences are additively separable across goods. For a given driver and reference-dependent model, we fix whether preferences are additively separable across goods, a model of reference points, and the form of reference-dependence. We then check WARP for the entire dataset for a neoclassical rationalization or, with additive separability across goods, Varian's (1983, Theorem 6) analogous conditions. If the neoclassical model fails, we find the pass rate by finding the largest subset of observations that satisfy WARP. For reference-dependent models, we next partition the data into gain-loss regimes and check WARP regime by regime.[23] We then find regimes' pass rates, construct Afriat utility functions for each regime using its largest set of rationalizable observations, use the Afriat utility functions to compute loss costs, check Proposition 5's inequalities (11) along the budget line, and compute the model's pass rate.[24] We compute the Selten areas

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[21] We calculate the area by numerical (Monte Carlo) integration over the budget constraints. New sets of choices that satisfy the budget constraints are repeatedly drawn and the conditions of interest are tested for each draw. The area is the proportion of those draws that satisfy the conditions. The area estimate converges as the square root of the number of draws. We draw until the uncertainty of the estimate is confined to the fifth decimal place.

[22] Selten's axioms are: (i) *monotonicity* $m(1,0) > m(0,1)$; (ii) *equivalence of trivial theories* $m(1,1) = m(0,0)$; and (iii) *aggregability* $m(\lambda r_1 + (1-\lambda) r_2, \lambda a_1 + (1-\lambda) a_2) = \lambda m(r_1, a_1) + (1 - \lambda) m(r_1, a_1)$ for $\lambda \in [0,1]$.

[23] Because we only need WARP rather than GARP, this is easily implemented for the non-additively separable model using R's `igraph` package. Details and code are contained in our Replication files.

[24] Condition (11) involves the entire Afriat regime utility functions, but they are finitely parameterized by the U_t^g and λ_t^g calculated in the first step and the $\{\boldsymbol{p}_t, \boldsymbol{q}_t, \boldsymbol{r}_t\}$. This involves a finite number of inequalities in a finite number of variables, and remains computationally feasible even in large datasets.

similarly, via Monte Carlo simulation with repeated random sampling as in Beatty and Crawford (2011).

Given the gap between sufficient and asymptotically necessary conditions (Propositions 5 and 6), we bound the pass rates and Selten measures as follows. To find the upper bound on the pass rate for a reference-dependent model we use the necessary condition that the data within each gain-loss regime must satisfy WARP within the regime, with no restriction to Afriat regime utilities or (11). We then calculate the pass rate for each regime and sum them to give an overall upper bound on the pass rate. To find the lower bound on the pass rate for a reference-dependent model we search across regimes for the maximal subset of the data that satisfies both the WARP condition within each regime and (11) across regimes. The upper bounds must be higher or at least no lower for the reference-dependent models, because WARP within each regime is less stringent that WARP for the full sample. The lower bounds, however, can go either way: Requiring WARP only within regimes helps the pass rate but requiring (11) and Afriat regime utilities hurts the pass rate.[25]

5.5 Main Results

Our main focus is comparing the neoclassical model with reference-dependent models. The latter all nest the neoclassical model, and must therefore have higher, or at least no lower, pass rates; but their Selten measures could be higher or lower, depending on their flexibility.[26] Our comparisons of Selten measures control for the extra flexibility of a given reference-dependent model, but *not* for considering different reference-point models for each driver, which would be computationally difficult. With that specification search in mind, we avoid arbitrarily favoring reference-dependent models by focusing on model comparisons that are independent of the reference-point model, which comparisons are surprisingly unambiguous.

[25] These arguments ignore the choice of rationalizing regime preferences for drivers who are reference-dependent on some but not all dimensions, or for those who are neoclassical. However, Propositions 1 and 5 continue to hold, mutatis mutandis, for preferences that are reference-dependent on less than the full range of dimensions; and the arguments extend straightforwardly to such cases.

[26] The is one exception here. For the experience-based reference point model we always lose one observation (or two in the case of lag models which condition on something) due to the construction of the lag. This can result in a higher pass rate for the neoclassical model. For example, driver 1's non-separable neoclassical pass rate is $\frac{36}{39}$. For the reference-dependent model in which the reference point is earnings last shift, we lose one observation due to the lag. This is subtracted from both the numerator and denominator and so the upper bound on the pass rate becomes $\frac{35}{38}$ which is less.

The complete sets of pass rates and Selten measures are reported in Online Appendices D and E, which cover non-additively separable and additively separable models respectively.

5.5.1 Additive separability across goods

In the literature on reference-dependent models, additive separability across goods has almost always been assumed, but it is not an intrinsic feature of such models. Figures 6 and 7 illustrate the densities of pass rates and Selten measures, estimated using rectangular kernel functions with supports and bandwidths corresponding to the bounds reported in Appendices D and E. The variation is across reference point models and drivers: 21 driver × 6 reference-point models × 3 different cases in which the reference point is operational, making 378 models represented for each density.

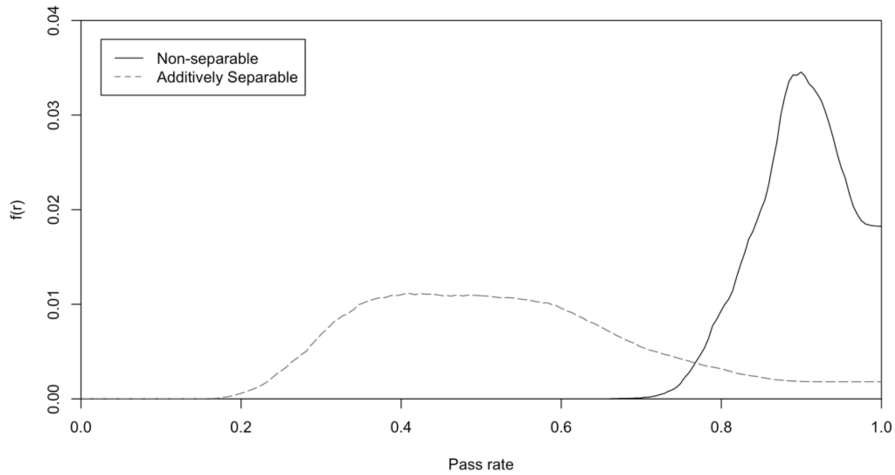
Figure 6's densities of pass rates for reference-dependent models show that models that relax additive separability across goods fit better than models that impose it, as they must. For neoclassical models, the mean pass rate for non-additively separable models is 0.9209 while the mean pass rate for additively separable models is 0.8175.

Figure 7's densities of Selten measures for reference-dependent models compare models that relax additive separability and models that impose it, controlling for their differences in flexibility. The heavy left tail for non-additively separable models (solid black line) reflects its greater flexibility. But the significant mass to the right shows that non-additively separable models have better overall predictive success. For additively separable reference-dependent models the density of Selten measures is centered close to zero, so that on average the model performs only slightly better than random. For neoclassical models the difference is small: The mean Selten measures are 0.3974 and 0.3965 for non-additively separable and additively separable models, respectively. But reference-dependent models that assume additive separability across goods perform worse than their counterparts that relax additive separability across goods. Accordingly, we set them aside from further consideration; the details are given in Online Appendices D and E.

5.5.2 Models of reference points

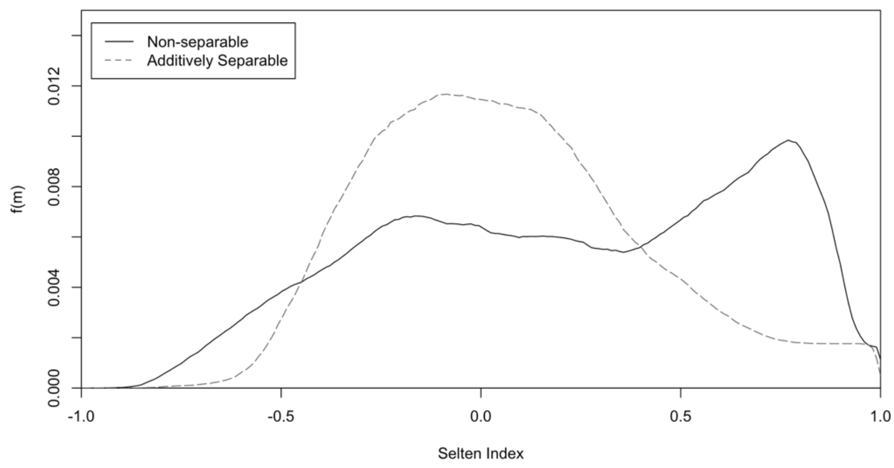
Regarding how reference points are modelled, Tables 2 and 3 give alternative models' mean pass rates and Selten measures, averaged over drivers and specifications for each class of model. The tables give simple counts of the number of drivers for whom a given class of reference-dependent model is the best. "Best" is defined in two alternative ways: The first panel ranks models by the mid-point of the lower and upper bounds, while the second ranks models by the lower bounds only—a best worst-case measure of performance.

Figure 6: Densities of pass rates for reference-dependent models



Notes: Smoothed kernel density estimates using rectangular kernel functions with bandwidths corresponding to the bounds on the pass rate illustrated in Appendices B & C.

Figure 7: Densities of Selten measures for reference-dependent models



Notes: Smoothed kernel density estimates using rectangular kernel functions with bandwidths corresponding to the bounds on the Selten measures illustrated in Appendices D and E.

Table 2. Summary of pass rates for across reference-point models that relax additive separability across goods

<table>
<tr><th></th><th colspan="4">Counts: best by mid-point</th><th colspan="4">Counts: best by lower bound</th></tr>
<tr><th></th><th>Hours</th><th>Earnings</th><th>Both</th><th>Mean</th><th>Hours</th><th>Earnings</th><th>Both</th><th>Mean</th></tr>
<tr><td>The leave-one-out mean.</td><td>12</td><td>13</td><td>11</td><td>12.00</td><td>14</td><td>15</td><td>15</td><td>14.67</td></tr>
<tr><td>The leave-one-out mean | rain.</td><td>11</td><td>11</td><td>10</td><td>10.67</td><td>15</td><td>15</td><td>16</td><td>15.33</td></tr>
<tr><td>The leave-one-out mean | night/day.</td><td>13</td><td>12</td><td>12</td><td>12.33</td><td>14</td><td>15</td><td>15</td><td>14.67</td></tr>
<tr><td>The last shift.</td><td>10</td><td>10</td><td>10</td><td>10.00</td><td>9</td><td>7</td><td>6</td><td>7.33</td></tr>
<tr><td>The last rainy/dry shift.</td><td>8</td><td>8</td><td>8</td><td>8.00</td><td>8</td><td>9</td><td>9</td><td>8.67</td></tr>
<tr><td>The last night/day shift.</td><td>9</td><td>10</td><td>9</td><td>9.33</td><td>10</td><td>8</td><td>8</td><td>8.67</td></tr>
<tr><td>Mean</td><td>10.67</td><td>10</td><td>10.39</td><td></td><td>11.5</td><td>11.5</td><td>11.56</td><td></td></tr>
</table>

Table 3. Summary of Selten measures across reference-point models that relax additive separability across goods

<table>
<tr><th></th><th colspan="4">Counts: best mid-point</th><th colspan="4">Counts: best lower bound</th></tr>
<tr><th></th><th>Hours</th><th>Earnings</th><th>Both</th><th>Mean</th><th>Hours</th><th>Earnings</th><th>Both</th><th>Mean</th></tr>
<tr><td>The leave-one-out mean.</td><td>8</td><td>7</td><td>7</td><td>7.33</td><td>8</td><td>8</td><td>8</td><td>8</td></tr>
<tr><td>The leave-one-out mean | rain.</td><td>4</td><td>4</td><td>3</td><td>3.67</td><td>6</td><td>6</td><td>7</td><td>6.33</td></tr>
<tr><td>The leave-one-out mean | night/day.</td><td>4</td><td>2</td><td>3</td><td>3</td><td>4</td><td>5</td><td>5</td><td>4.67</td></tr>
<tr><td>The last shift.</td><td>7</td><td>8</td><td>7</td><td>7.33</td><td>8</td><td>7</td><td>6</td><td>7</td></tr>
<tr><td>The last rainy/dry shift.</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td><td>3</td><td>2.67</td></tr>
<tr><td>The last night/day shift.</td><td>5</td><td>6</td><td>5</td><td>5.33</td><td>6</td><td>5</td><td>5</td><td>5.33</td></tr>
<tr><td>Mean</td><td>4.83</td><td>4.5</td><td>4.78</td><td></td><td>5.67</td><td>5.67</td><td>5.67</td><td></td></tr>
</table>

In Table 2's pass rates, models with expectations-based reference points dominate. Using the mid-point as a summary of the bounds, the expectations-based model in which the leave-one-out mean of hours is used as the reference point has the highest, or equal-highest, pass rate for 12 of the 21 drivers. That model also has a lower bound on the pass rate which is the highest or equal-highest lower bound for 14 of the 21.

In Table 3's Selten measures, the picture is more nuanced. Looking across columns, the form of reference-dependence (hours, earnings, or both) matters relatively little. Looking across rows, the class of model (expectations- versus experienced-based reference points) matters more. The mean Selten measure for models with reference-dependence in both earnings and hours is slightly higher than for models with reference-dependence in earnings only or hours only, and still higher than for the neoclassical model. The recent experience-based model with reference points determined by hours and earnings on the last day or night shift has the highest or equal-highest Selten measure for 5 of the 21 drivers. Overall, unconditional models perform better than models that condition on weather or time of day. To sum up, making allowance for flexibility, reference-dependent models have an advantage over the neoclassical model for many drivers. Unconditional, expectations-based reference point models perform best, but the results are almost independent of the model of reference points. From now on, for reference-dependent models we assume unconditional expectations; the details are given in Online Appendices D and E.

5.5.3 Comparing reference-dependent models and the neoclassical model

Our final comparison is between reference-dependent models and the neoclassical model. As our previous comparisons suggest, we focus on models that relax additive separability across goods and on unconditional expectations-based models of reference-dependence. Figure 8 shows radar plots for such models with panels for alternative forms of reference-dependence: hours, earnings, or both hours and earnings. Each panel has a “spoke” for each driver, with the plot centered at the lowest possible pass rate of 0 and with outer rim at the highest possible rate of 1. The solid line traces the pass rates for the neoclassical model across drivers. The shaded areas depict the bounds on the pass rate for the reference-dependent models. The reference-dependent models’ pass rates are higher than the neoclassical models’ for all drivers, as they must be. For many drivers the bounds point identify the pass rate, with the data satisfying both the necessary and the sufficient condition.

Figure 9 shows the analogous radar plots for Selten measures. The plots are now centered at the lowest possible Selten measure of -1, with outer rims at the highest possible measure of 1. The solid lines now trace the measures for the neoclassical model across drivers. For some drivers and forms of reference-dependence (with a + on the driver index) the neoclassical model’s measure is at or below the lower bound for the reference-dependent model. For others (with a -) the neoclassical model’s measure is at or above the upper bound for the reference-dependent model. For others (with a +-) the bounds are tight but the models are tied. And for a few others (with no mark on the index), the bounds do not yield an unambiguous comparison.

Overall, the plots show that qualitative model comparisons differ across the form of reference-dependence only slightly, for two drivers (1 and 19). Accordingly we focus on the the model with reference-dependence in both hours and earnings, with plot in the right-most panel. In this panel, as in the other two, neither model has uniformly better predictive success. The reference-dependent model unambiguously (given the bounds) outperforms the neoclassical model for 6 of the 21 drivers (1, 10, 16, 18, 20, and 21), all of whom are in the "original" 15 with comparatively large samples that Farber and Crawford and Meng analyzed. The neoclassical model unambiguously outperforms the reference-dependent model for 9 of the 21 (2, 3, 5, 6, 9, 11, 13, 14, and 15) but only 4 of the original 15 (2, 5, 9, and 14).[27] The bounds are tight but the models are tied for 2 of the 21 (4 and 8), both in the 15. The bounds do not yield an unambiguous comparison for 4 drivers (7, 12, 17, and 19), 3 (7, 12, and 19) in the original 15. Thus, penalizing reference-dependent models for their greater flexibility does not by any means make them meaningless in Samuelson's sense: They appear to provide a clearly useful supplement to the neoclassical model for roughly half of Farber's drivers, particularly for those with comparatively large samples.

6. CONCLUSION

This paper presents a nonparametric analysis of the theory of consumer demand and labor supply with reference-dependent preferences. Our nonparametric model is in the style of Kőszegi and Rabin's (2006) parametric model, but does not directly impose their assumptions on functional structure or Farber's (2005, 2008) and Crawford and Meng's (2011) assumptions on functional structure and form. Our goal is to learn to what extent the refutable implications of such models stem from reference-dependence, constant sensitivity, or loss aversion per se, or are artifacts of the strong ancillary assumptions that have so far been maintained in the theoretical and empirical literatures.

[27] Recall that 5 of the 6 drivers with small samples that Farber and Crawford and Meng omitted satisfy WARP and have perfect fits for the neoclassical model, which model must then have a higher Selten measure than its reference-dependent counterpart due to the latter's greater flexibility. For these drivers our model comparison is somewhat less informative.

Figure 8. Pass rates; unconditional expectations-based, non-separable models

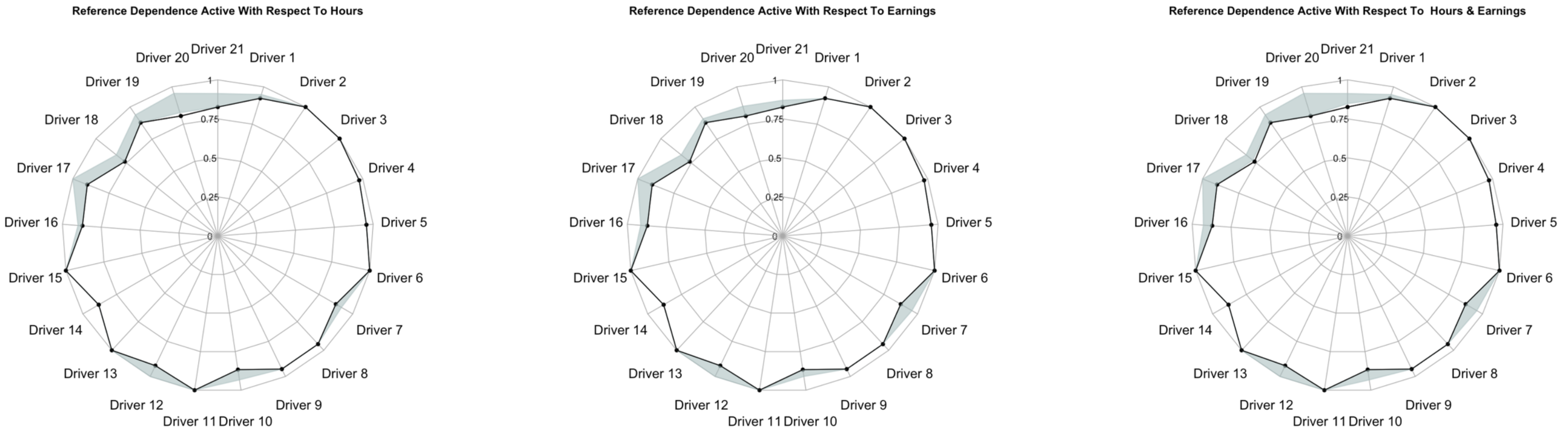
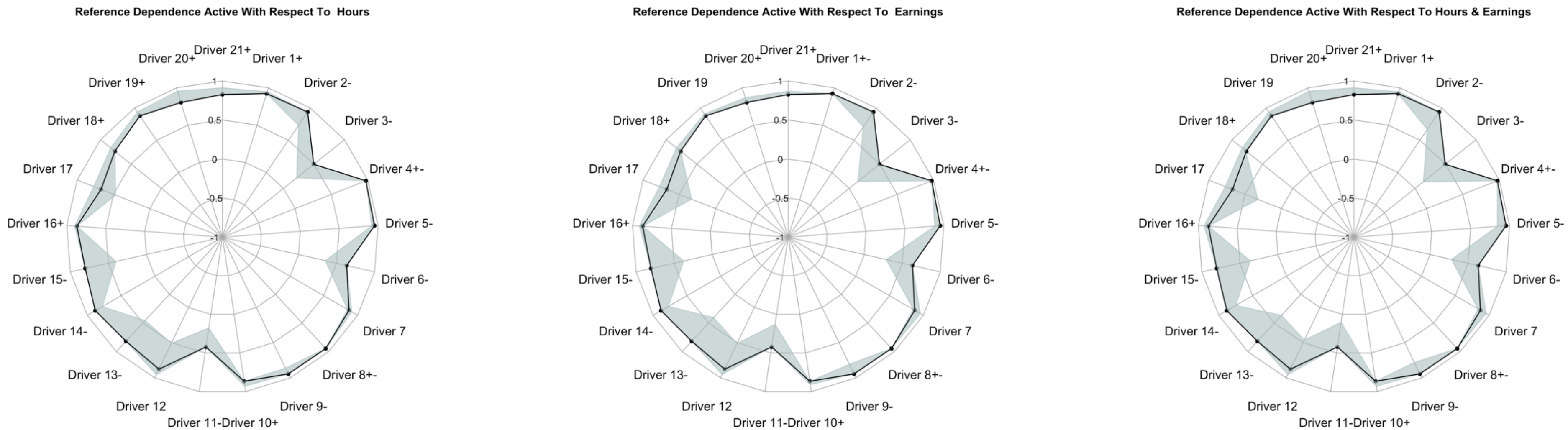


Figure 9. Selten measures; unconditional expectations-based, non-separable models



Proposition 1 characterizes preferences that have additively separable consumption and gain-loss utility components, as in Kőszegi and Rabin (2006), and that satisfy constant sensitivity and a natural assumption of joint continuity in the consumption bundle and reference point. The characterization yields a class of preferences that nest Farber's, Kőszegi and Rabin's, and Crawford and Meng's models with constant sensitivity, while relaxing their knife-edge restrictions on how preferences vary across gain-loss regimes and their assumption of additive separability across goods, both of which are important in our illustrative re-analysis of Farber's data.

Propositions 2 and 3 show that unless reference points are observable or precisely modelable and sensitivity is constant, the hypothesis of reference-dependent preferences has few or no nonparametrically refutable implications. Propositions 4 to 6 show, however, that when reference points are observable and sensitivity is constant, the hypothesis of reference-dependent preferences has refutable implications that can allow a rationality-based, parsimonious rationalization of a consumer's choices, even if they violate GARP. Proposition 5 gives sufficient and asymptotically necessary conditions for the existence of reference-dependent preferences that rationalize choice, based on the Afriat rationalizing regime utilities. Those conditions are more complex than the GARP coindition Afriat's Theorem provides for the existence of a neoclassical rationalization, but they make it computationally tractable, even in large datasets, to test for consistency of the data with reference-dependent preferences and to recover rationalizing preferences when they exist.

Constant sensitivity and modelability of reference points play important and possibly unexpected roles in our analysis. The literature seldom stresses the importance of modelability (Kőszegi and Rabin 2006 and Crawford and Meng 2011 are exceptions), but it is required for a reference-dependent model of consumer demand to have *any* nonparametrically refutable implications. Constant sensitivity is also needed for useful nonparametric implications.

We illustrate the methods our results suggest by using them to reconsider Farber's (2005, 2008) and Crawford and Meng's (2011) structural econometric analyses of New York City cabdrivers' labor supply nonparametrically. We allow unrestricted driver heterogeneity and compare alternative models of reference points. The assumption of additive separability across goods, maintained in all previous work in this area, is strongly rejected. Relaxing it, even though the GARP condition for a neoclassical rationalization is violated for most of Farber's drivers, reference-dependent preferences can rationalize most choices for many of them. Model comparisons using Selten and Krischker's (1983) measure of predictive success to control for reference-dependent models' greater flexibility (see also Selten 1991 and Beatty and Crawford 2011) suggest that for roughly half of Farber's drivers, reference-dependent models' fits are sufficiently better to justify their greater flexibility.

We hope that our analysis shows that reference-dependent models of consumer demand can provide a tractable and empirically useful addition to the neoclassical consumer demand toolkit.

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