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THE SUPPLY AND DEMAND FOR FEDERAL RESERVE DEPOSITS

BY

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The Supply and Demand for Federal Reserve Deposits

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1. Introduction

It has been nearly 20 years since Christopher Sims published his incisive critique of the manner in which builders of large-scale econometric models claimed to connect their models to reality. Sims raised powerful objections to the casual, even haphazard way in which some applied econometricians were implicitly restricting the dynamic interactions between variables in order to come up with structural labels for estimated historical correlations. The critique was particularly compelling because Sims had a constructive alternative to propose-- leave the dynamics completely unrestricted and simply estimate the reduced form of the system of structural equations through a vector autoregression.

Unfortunately, as cogently observed by Cooley and LeRoy (1985), many of those who followed Sims's lead ended up implicitly putting structural labels on the reduced-form equations, for example, by equating the error the VAR makes in forecasting the money supply with a "shock" or "innovation" to monetary policy.

It has been over a decade since Ben Bernanke (1986), at this conference, and Blanchard and Watson (1986) and Sims (1986) himself in related work, proposed a legitimate basis for putting such labels on VAR innovations and thereby understanding what the dynamic correlations captured by a VAR might mean. This approach calls for an explicit acknowledgement that some sort of exclusion restrictions are necessary in order to know which linear combination of the VAR innovations could plausibly be called a policy shock. Bernanke shared Sims's distrust of the detailed, ad hoc restrictions on dynamics cavalierly adopted by an earlier generation, and followed Sims in leaving the dynamics completely unrestricted. This "structural VAR" approach calls for using the absolute minimum of solely contemporaneous exclusion restrictions, implemented with great caution and explicit acknowledgement of potential limitations, in order to achieve identification, or, in Sims's words, in order to connect the statistical models with reality.

As one surveys empirical macroeconomic research today, and the papers of this conference in particular, it appears that the Bernanke–Sims approach has now become the accepted standard for reporting and interpreting macroeconomic correlations. Such unanimity is a remarkable testimony to the vision of these early scholars. However, although there is widespread agreement as to the appropriateness of the method, no one seems to find the results all that persuasive, or at least, they are not in agreement as to what the results in fact persuade us of. For example, Christiano, Eichenbaum, and Evans (1996, p. 33) conclude that "these responses accord to a striking degree with conventional views about how monetary policy shocks affect the economy." Leeper, Sims and Zha (1996, p. 2) are equally adamant that "only a modest portion (in some cases, essentially none) of the variance in output or prices in the United States since 1960 can be attributed to shifts in monetary policy." By contrast, Pagan and Robertson (1995, p. 52) are unsure about what the results prove, and proud of it: "The models do not seem to be very robust to data coming from the 1980s; The implied structural models can sometimes be implausible; The estimation procedures often rely on weak information and ... the long–run multipliers can be contrary to a priori beliefs."

I would argue that the reason this methodology has failed to produce more compelling results is precisely the concern raised by the original advocates of VARs— convincing identifying assumptions are hard to come by. In saying this, I am going beyond the current consensus view that identifying assumptions must be made with great care. Instead, I regard the standard exercise— fitting a vector autoregression to a group of key macro variables and hoping to come up with identifying assumptions on the basis of which to interpret the residuals— as inherently hopeless. The reason is that valid exclusion restrictions are much too rare a commodity for us to expect to be able to come up with $n(n - 1)/2$ of them in an arbitrary set of n macroeconomic aggregates.

I instead propose the following as a constructive guide to empirical research in macroeconomics.

(1) In order to be able to give historical correlations a causal interpretation, one must start from the claim that an observable component of the variation in the causal variables is the outcome of a natural experiment, in which the causal variable was randomly shifted through a process independent of the system being studied. The variables one chooses to study and the questions one asks about them are predicated on first recognizing and establishing the validity of this natural experiment.

(2) The economic meaning of the structural equations being investigated should be explicitly stated by the researcher at the outset— exactly whose behavior or what market outcome is being described by this equation? The full range of institutional details and economic understanding of what such a relation should look like should then be drawn on for purposes of evaluating the plausibility of the empirical estimates of the parameters that are supposed to capture that relation.

(3) The model should be subjected to full statistical corroboration. Coefficients should be tested for stability, models should be evaluated post-sample, overidentifying assumptions should be tested, and refutable predictions of the framework should be avidly sought and investigated.

This is a more difficult recipe for empirical research than fitting a VAR to an arbitrary set of variables, but I believe that we will ultimately learn much more from it.

This paper is a constructive example of how such research might proceed in practice. The primary instrument of monetary policy is manipulation of the supply of Federal Reserve deposits through open market operations. The immediate impact of these open market operations is determined by the adjustment of the federal funds rate, which measures the overnight opportunity cost of Federal Reserve deposits. The magnitude of this adjustment is governed by how much the funds rate must move in order to equate the demand for deposits with the new level of supply. The question studied in this paper is therefore, what factors determine the supply and demand for federal reserve deposits?

A considerable part of this exercise consists of describing the dynamic interactions between various magnitudes from the daily balance sheet of the Federal Reserve system. On one level, the study might be described as simply a VAR fit to these variables with structural labels placed on the various innovations. However, the exercise differs from most other estimated VAR models in several respects. First, literally hundreds of exclusion restrictions are imposed, and all are explicitly tested. Some of these exclusion restrictions arise from the idea that certain calendar events–– for example, the difference between a Wednesday that is the last day of a two–week reserve maintenance period and a Wednesday that is in the middle of a two–week reserve maintenance period–– matter for some economic decisions and not others. Detailed modeling of deterministic periodicities is accordingly given a lot of attention in this study. Other exclusion restrictions arise from the claim of strict econometric exogeneity, that certain decisions are completely unaffected by other variables, and that some variables are beyond the control of particular economic agents.

The fruit of this inquiry is estimates for a particular structural model of how the banking system reacts to a temporary shock to the supply of Federal Reserve deposits. We find that, unless the shock occurs on a settlement Wednesday or the last day of a quarter, banks do not go to the discount window to replace temporary disruptions in the supply of Federal Reserve deposits. Instead, they simply make do with fewer reserves on that day. Even though a reserve requirement does not bind on that day, banks are only persuaded to hold fewer reserves on that day if there is an increase in the federal funds rate. We estimate that a temporary $1 billion reduction in the supply of reserves on such days will result in an increase in the federal funds rate of 2.6 basis points. By contrast, on settlement Wednesdays or the last day of the quarter, banks do replace some of the lost reserves through the discount window, so that the net effect on the supply of reserves is smaller. Even so, the consequence of the disruption for the federal funds rate is much bigger, with a a $1 billion reduction in the supply of reserves on such days typically resulting in a 6.6–basis–point increase in the federal funds rate. The paper proposes a number of natural experiments in which the supply of reserves has been exogenously reduced and in which one finds statistically significant corroboration of this liquidity effect (lower reserves raise the federal funds rate). The observed outcome of these experiments is consistent with the prediction from the structural estimates.

We begin in Section 2 with a description of the data used in this study.

2. A consolidated balance sheet for the Federal Reserve

The balance sheet of the Federal Reserve system is reported for selected days in publicly available sources such as Table 1.18 in the Federal Reserve Bulletin of June 1992. We were able to obtain for this project corresponding values for the balance sheet for every business day during 1992 to 1994, or a sample of 756 observations.![1]

Most of the individual entries in the Fed's balance sheet are of limited interest, and for our purposes it suffices to work with a consolidated version of the balance sheet. Table 1 groups individual items into six main categories, described in detail below.

Securities and other net assets (S_t)

By far the most important asset of the Federal Reserve is securities acquired through open market purchases. For purposes of this study we make no distinction with respect to maturity or whether the debt was originally issued by the U.S. Treasury or by a federal agency. We also do not distinguish between securities held outright and those acquired through repurchase agreements. Even though the latter will be transferred out of the Fed's assets within a few days, if the securities are held by the Fed on day t, then they are included in our measure S_t for that day.

Another important asset category for the Fed is assets denominated in foreign currencies. Although this category responds to quite different factors than do typical open market operations, we nevertheless include it in our measure of S_t. The reason is that we are not interested in open market operations that simply sterilize foreign currency intervention. Note that if the Fed acquires foreign currency and sterilizes this with an open market sale, the value of S_t would be unchanged for the day.

There are also some slow-moving assets of the Fed such as the gold certificate account and bank premises. These are of little interest and we choose to include them in S_t. We likewise subtract miscellaneous items from

!I am deeply indebted to Joe Dziwura for his considerable efforts to help me obtain these data.

the liabilities side of the balance sheet (which for a private firm would be viewed as the Fed's net equity), before arriving at the value S_t, which we designate as securities and other net Federal Reserve assets.

<u>Discount window loans</u> (L_t)

The second asset category that we are interested in is loans the Fed has made to private banks through the discount window. On a typical day (such as that reported in Table 1), this would be under $200 million, and would rarely exceed $1 billion. Although the magnitude of discount borrowing is small, we will argue in this paper that fluctuations in L_t, or the degree to which banks are forced to borrow directly from the Fed, is a key element in understanding exactly how open market operations affect the banking system.

<u>Net float</u> (F_t)

The Fed provides some check–clearing and related services for private banks. At any given time the Fed has received checks and other items which it is in the process of collecting and transporting to the banks on which the checks are written. The Fed does not actually receive payment for the checks, that is, the Fed does not debit the reserve account of the bank, until the checks are delivered. Possession of such undelivered checks represents an asset of the Fed, denoted "items in process of collection."

Similarly, the Fed does not immediately credit the bank which presents the check for deposit, but promises to do so within one or two business days. The quantity of presented checks for which the Fed expects to give credit is a liability of the Fed referred to as a "deferred credit item."

The difference between these two numbers, "items in process of collection" minus "deferred credit items," is called "Federal Reserve float." When this is positive, it means there is a check for which the receiving bank's account has been credited (so it's no longer a deferred credit item) but the paying bank's account has not yet been debited (so it's still an item in process of collection). Thus both the paying and receiving bank are counting these funds as deposits with the Federal Reserve. An increase in float simultaneously creates an asset for the Federal Reserve (the checks waiting to be collected) and a liability (the double-counting of Federal Reserve deposits).

On a typical day float is small, but it can exhibit large fluctuations when storms disrupt the normal process of check collection and delivery.

Float can also arise from incomplete or misdirected wire transfers. For this reason, float is sometimes associated with other anomalies in the balance sheet. The most dramatic example of this occurred on September 9, 1992, when float jumped to an astonishing $19.6 billion, and yet none of the four categories of holders of reserves— depository institutions, the U.S. Treasury, foreign official accounts, or others— were credited with holding these reserves; in other words, the balance sheet didn't balance on this day. Moreover, the biggest factor in producing this tremendous value for float was a negative value for deferred credit items of $12.5 billion.

We have chosen to deal with these rare anomalies by first creating a balance sheet that is balanced by construction, namely by defining any Federal Reserve deposits that are not held by depository institutions or by the U.S. Treasury to be deposits that are held by other institutions." We then subtract this measure of deposits held by other institutions from float to come up with a magnitude we call "net float." Our measure of net float thus

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"That is, we replace series [25] (foreign official accounts) plus series [26] (other deposits) with series [22] (total deposits) minus series [23] (depository institutions) and series [24] (U.S. Treasury— general account). There are only a handful of observations for which [25] + [26] is a different number from [22] - [23] - [24], with September 9, 1992 being by far the most dramatic.

captures only those changes in float that actually show up as reserves held by depository institutions or the U.S. Treasury. The effect of this adjustment is that the wild values for September 9, 1992 get completely netted out.

Federal Reserve Notes (N_t)

The biggest single item on the liability side of the Fed's balance sheet is Federal Reserve notes, which tracks the total volume of currency outstanding.

Federal Reserve Deposits held by depository institutions (D_t)

This denotes the total dollar value at the end of the day that banks hold in accounts with the Federal Reserve. When the Fed purchases $100 million in Treasury bills on the open market, S_t goes up by $100 million on the asset side of the Fed's balance sheet and the Fed pays for the securities by crediting banks with an additional $100 million in D_t on the liability side. If banks borrow $100 million from the Fed at the discount window, then L_t goes up by $100 million on the asset side and the same sum is credited to D_t on the liability side.

Banks can obtain currency from the Fed by surrendering Federal Reserve deposits in return for currency. In such a transaction, D_t would go down by the same amount that N_t goes up, and total assets or liabilities of the Fed would be unchanged.

Federal Reserve deposits held by the U.S. Treasury (U_t)

The U. S. Treasury also maintains an account with the Federal Reserve. When a member of the public writes a check to the Internal Revenue Service, the Fed debits the account of the bank on which the check is drawn (so that D_t goes down) and credits the Treasury's account (so that U_t goes up by the same amount). Again such a transaction has no effect on the total assets or liabilities of the Federal Reserve.

Summary of the consolidated Federal Reserve balance sheet

We have simplified the Fed's balance sheet into three categories of assets,

S_t = securities and other net Federal Reserve assets

L_t = discount window loans

F_t = net float,

and three liabilities,

N_t = Federal Reserve notes

D_t = Federal Reserve deposits held by depository institutions

U_t = Federal Reserve deposits held by the U. S. Treasury.

These six magnitudes are related by the accounting identity

$$S_t + L_t + F_t = N_t + D_t + U_t \tag{1}$$

which holds exactly for every day t in the sample.

3. Econometric analysis in the presence of accounting identities

The purpose of this paper is to develop a complete description of the dynamic links between the six variables in equation (1) for purposes of understanding the factors that determine the supply and demand for Federal Reserve deposits. Before getting into the details, however, it is helpful to make some preliminary observations about how the accounting identity (1) should be incorporated in such an analysis.

One important principle can best be understood from the following example. Imagine generating n different time series $(y_{1t}, y_{2t}, \ldots, y_{nt})$ as follows:

$$y_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_i + \varepsilon_{it} \qquad \text{for } i = 1,2,,\ldots,n-1 \tag{2}$$

$$y_{nt} = -(y_{1t} + y_{2t} + \cdots + y_{n-1,t}). \tag{3}$$

Here $\mathbf{x}_{it}$ is a vector of explanatory variables that are important for determining y_{it}. Note that these n variables satisfy the accounting identity

$$\sum_{i=1}^{n} y_{it} = 0 \qquad (4)$$

for all t by construction. However, the existence of the accounting identity (4) does not in any way restrict any of the individual relations in (2). In particular, some of the variables y_{it} could exhibit time trends while others do not, some could be integrated and others not, and y_{it} could be generated completely independently of y_{jt} for $i,j = 1,\ldots,n-1$ and $j \neq i$. What is true is that if a variable z_t matters for y_{it} for some $i = 1,2,\ldots,$ or n, then z_t must also matter for at least one other variable. In the system described by equations (2) and (3), the vector $\mathbf{x}_{it}$ must affect both y_{it} and y_{nt}. This means, for example, if one of the n variables exhibits a deterministic time trend, then at least one other variable must also exhibit a time trend.

This example also illustrates another important principle of analyzing variables that are related by an accounting identity— once one has specified the determinants of n – 1 of the variables in the system, then the nth equation is redundant and contains no additional statistical information. One philosophy of treating such systems is simply to drop one of the variables as redundant and model the remaining n – 1. While this is valid in principle, in practice one must exercise some care as to which variable to treat as the "residual;" Bewley (1986) offers an excellent discussion of the issues involved. For example, suppose that data were really generated by the system (2)–(3) but one specified y_{1t} as the residual and tried to estimate equations for variables y_{2t} through y_{nt}. In principle, one could find what would amount to the identical process followed by the n variables, but the representation would be less parsimonious and harder to recognize than if it were written in the form of (2)–(3).

In the application of this paper, we will argue for a particular economic interpretation of the vector time series process that generated the six variables of interest. Specifically, we will claim that U_t, N_t and F_t are generated by factors totally exogenous with respect to the goals of monetary policy or the demand by banks for reserve deposits; a variety of institutional and econometric evidence will be presented in support of this claim. We will argue that monetary policy, specifically the Federal Reserve's trading desk, responds to these exogenous shocks through manipulation of the magnitude it controls directly, namely S_t. Private banks take the disturbances to U_t, N_t, F_t, and S_t as given and make a choice as to how much to borrow at the discount window L_t in response. This choice then determines banks' reserves D_t through the accounting identity (1). Thus we are treating D_t as the "residual" variable in the system. This does not mean that it is the least important variable or that we have little to say about its determinants. Just the opposite is the case— the other variables are of interest primarily <u>because</u> they matter for banks' reserves, and the implicit empirical description of the determinants of D_t reported in this paper is the most detailed and involved of any of the relations investigated— <u>every</u> variable investigated here has some effect on D_t.

The subsequent sections present our econometric analysis of the determinants of each of the individual elements of the consolidated Fed balance sheet.

4. Federal Reserve deposits held by the U.S. Treasury

Figure 1 plots the daily values for U_t, the Treasury's balance with the Fed. The behavior of this series over 1989–1991 was described in detail in Hamilton (1997), and the appearance of the new data for 1992–1994 in Figure 1 is quite similar. In particular: (1) the Treasury balance typically ends each day within a billion dollars of the $5 billion target that the Treasury and the Fed are trying to hit; (2) during times of heavy tax receipts, the usual Tax and Loan Accounts that the government employs to achieve this target reach their capacity, and the Treasury balance can shoot dramatically upward; (3) these bulges in the Treasury balance usually disappear quickly as a result of the heavy fiscal expenditures that come with the start of a new month.

As a first step, I re–estimated the model proposed by Hamilton (1997), the only differences being the sample periods and the fact that the previous study used maximum likelihood to control for ARCH and outliers, whereas, given the large number of variables to be analyzed here, the current study simply uses OLS for all estimation. The original estimates and the new results are reported together in Table 2. The earlier specification modeled the filling of the Tax and Loan Accounts through a threshold autoregression; if the previous day's Treasury balance exceeded $8 billion, then a different constant term and autoregressive coefficient are estimated (the coefficients on U_{1t} and $U_{1t}U_{t-1}$, respectively— see Table 3 for a summary of the notation used in this and all subsequent tables). The tendency of the start of a new month to kick the process out of the tax–bulge regime is captured by U_{2t} and $U_{2t}U_{t-1}$. Other variables allow for influences of the major tax collection periods (U_{3t}), a separate dummy for April (γ_{4t}), a Friday effect (ξ_{5t}), end–of–month effect (C_{2t}), and Social Security disbursements ($I[C_{1t}=3]$ and $I[C_{1t}=4]$).

The correspondence between the old and new parameter estimates is quite remarkable, given that there is zero overlap between the two data sets. All of the coefficients retain the same sign, and most are quite close in magnitude to the original estimates and remain highly statistically significant. The median absolute t–statistic of the set of new estimates reported in Table 2 is 3.2. Such stability of a detailed statistical model is rather rare in macroeconomics, and I would argue speaks to the soundness of the underlying research strategy, namely, let the specification be guided by a detailed understanding of the relevant institutions, exercise care to avoid simultaneous equations bias, and subject the model to rigorous specification testing before concluding it is correct.

Although the original specification performs very well, additional testing with the new data set suggested several ways it might be improved. First, the dummies for Social Security disbursement ($I[C_{1t}=3]$ and $I[C_{1t}=4]$ in Table 2) are no longer statistically significant, and so were dropped in the interests of parsimony. Second, the fourth lag of the Treasury balance (U_{t-4}) proved to be significant in the new data set, and so it was included in the specification that will be used here (see Table 4). Third, it is apparent from Figure 1 that the last half of December is also characterized by unusually high values for the Treasury balance. Thus the original indicator for the major tax collection periods, U_{3t}, which was defined as the second half of January, April, June, and September, was replaced for this study by U_{4t}, which covers the second half of January, April, June, September, and December. Modeling the December contribution to tax receipts is important for this study given the significant role of the Christmas season for some of the other balance sheet components of interest here. We use the same day as in Hamilton (1997) for determining when the second half of a month begins, namely, the day following the first Monday after the 15th of the month. This day is also strikingly important for the Treasury balance in the new data set, so we have also included a new dummy variable U_{5t} for this particular day during the major tax collection months. Finally, we find some suggestion of more general day–of–the–week effects than the simple Friday dummy used in the earlier analysis; the Treasury balance is often slightly higher on Monday and

Tuesday than other days of the week. The model used here allows a separate intercept for each day of the week.

How does one decide whether this is an adequate representation of the time series? Table 5 tests whether a variety of other potential indicators have been correctly excluded from our model of the Treasury balance. These tests cover over a hundred excluded variables and include all of the explanatory variables used in the various models estimated in this paper. In every case we accept at the 5% level the null hypothesis that the model of the Treasury balance reported in Table 4 has been correctly specified. Note that Table 5 adopts a general structure so that it can also be used to report evaluations of subsequent models as well, so that sometimes a subset or all of the variables described in a given row have already been included in any one model. When this is the case, the test reported in Table 5 refers only to those variables that are not already included in the model. Thus, for example, of the seven tax indicators listed in the tenth row of Table 5, only U_{3t}, the dummy for the major tax collection periods used in Hamilton (1997), was not already included in the model for the Treasury balance. Thus the hypothesis test for the first column of this row has only one degree of freedom, and is a test of the single restriction that U_{3t} does not belong. Equivalently (since the broader indicator U_{4t} is included), this is a test of the restriction that the Treasury balance displays the same kind of dynamics in the latter part of December as it does in the other tax collection periods. Although the p–value of 0.06 is close to rejecting the null hypothesis, we regard this as supporting the choice of including December with the other months rather than giving it a new separate dummy. All of the other null hypotheses in the first column of Table 5 are readily accepted.

We conclude that the description of the determinants of the Treasury's balance of Federal Reserve deposits offered in Hamilton (1997) is substantially confirmed— this magnitude is governed entirely by fiscal receipts and expenditures and the timing of when particular payments are made and checks clear. In particular, the size of U_t can be treated as strictly econometrically exogenous with respect to either monetary policy or banks' desired levels of borrowed or excess reserves.

The claim that U_t is strictly econometrically exogenous is refutable; if this series does not respond to open market operations or anything banks can choose, then lagged values of any of the other magnitudes in the Fed's balance sheet should not be useful in predicting the value of U_t. Table 5 indeed confirms that none of the other variables, either individually or taken as a group, "cause" in the definition of Granger (1969) the value of U_t. Note the role of the accounting identity in testing the latter hypothesis— if a regression already includes the value of S_{t-1}, L_{t-1}, F_{t-1}, N_{t-1}, and U_{t-1}, then including the value of D_{t-1} would be a redundant regressor since its value is an exact linear combination of the other five. Thus the null hypothesis described in the second to last row of Table 5 that nothing from the previous five days' worth of balance sheets helps predict the value of U_t other than four lags of itself, is actually a restriction that the 21 coefficients on the variables U_{t-5} and S_{t-j}, L_{t-j}, F_{t-j}, N_{t-j} for $j = 1,\ldots,5$ are all zero. Note that this restriction is readily accepted.

This evidence supports the first major identifying assumption for the estimation of structural models that will follow. Specifically, any contemporaneous correlation between U_t and other balance sheet terms should be interpreted as the response by the Fed or banks to the Treasury balance, rather than the other way around.

5. Federal Reserve Notes

Figure 2 plots the value for N_t, Federal Reserve notes outstanding. This series displays a strong upward trend, rising from $280 billion at the start of the sample to $380 billion by the end. Plotting the three years together as in Figure 2 highlights the importance of seasonal components as well. The most dramatic of these is a huge run–up in cash during the Christmas spending season each year that gradually gets returned to the Fed during January. A mini version of the same pattern can be seen with the other holidays.# There is a big bulge in cash prior to July 4, Labor Day, Thanksgiving, and, surprisingly, Columbus Day, and more minor increases prior to President's Day, Memorial Day, and Veteran's Day. The identical annual regularities seem to hold outside the sample as well; see for example the plots for 1986 to 1988 given in Meulendyke (1989, p. 142). The run–ups prior to the non–Christmas holidays presumably result from the extra use of cash for travel and leisure pursuits at these times. In addition, there is a strong day–of–the–week effect— cash outstanding tends to be higher at the beginning of the week than at the end. This weekly pattern is much more prominent for 1994 than the two earlier years.

Table 6 presents estimates of our statistical model for N_t. The model includes a time trend, ten lags of N_{t-j}, a dummy for December (the coefficient on $\gamma_{12,t}$), and day–of–the–week effects (ξ_{jt}) which are allowed to be different in 1994 from the rest of the sample. We also found a clear tendency for cash to be higher at the beginning and end of each month than in the middle; the

#The banking holidays for the U. S. are January 1, Martin Luther King Day (the third Monday in January), President's Day (third Monday in February), Memorial Day (the last Monday in May), July 4, Labor Day (first Monday in September), Columbus Day (second Monday in October), Veteran's Day (November 11, or the following Monday if November 11 falls on a weekend), Thanksgiving (the fourth Thursday in November), and Christmas (December 25, or the day after if December 25 falls on a Sunday).

coefficients on the day of the month (C_{1t}) and its square have t statistics near ten in absolute value. Cash goes up the closer one gets to an approaching holiday (the coefficients on H_{1t} and H_{2t}). Holidays of course must also interact with day–of–the–week effects; if Monday is a holiday, then whatever usually happens on Mondays must show up on the preceding Friday or the following Tuesday that week instead. We find the most striking confirmation of this in 1994, when the day–of–the–week effects are most dramatic— separate 1994 dummies for the day before a 1– or 3–day holiday (H_{5t} or H_{6t}) or the day after a 3–day holiday ($H_{6,t-1}$) are highly statistically significant. Since it is unclear a priori exactly how a typical weekly pattern should be disrupted by a holiday, and since the public's use of cash introduces a dramatic effect of the holiday itself, we have not tried to model the interaction between holidays and day–of–the–week effects in further detail. In the interests of parsimony we have instead just tried to capture what seem to be the most dramatic features of the data. We also pay particular attention to Christmas— the model captures the initial surge in cash prior to Thanksgiving (X_{4t} and X_{5t}), the build–up as Christmas approaches (the coefficients on X_{2t} and X_{3t}), plateau in the week around Christmas (X_{1t}), and subsequent decline (X_{6t} and X_{7t}).

As in the model for the Treasury balance, there is no doubt that the included variables are extremely important determinants of daily fluctuations in the quantity of Federal Reserve notes— the coefficients in Table 6 have a median absolute t statistic of five. Table 5 subjects this model to the same battery of specification tests as was used to evaluate the model of the Treasury balance. Overall the model does quite well, though more detailed analysis of the role of holidays might be justified.

What does this statistical model tell us about the nature of the economic forces that determine the quantity of Federal Reserve notes? Certainly the long–term upward trend is the outcome of monetary policy— the public's cash holdings could hardly have increased by $100 billion over these three years if the Fed had kept its securities holdings constant. For that matter, it would be impossible for cash holdings to expand and contract over Christmas to the degree exhibited in the data if there were not large open market purchases in December and corresponding sales in January. Nevertheless, it seems quite clear that it is the public's demand for cash at Christmas that causes this seasonal behavior of open market operations, rather than the other way around. Indeed, all of the variables in the model in Table 6, apart from the trend, are much more naturally interpreted as seasonal factors that influence the demand for cash which the Fed passively accommodates, rather than as regularities that for some reason are deliberately introduced by monetary policy. For example, if the day–of–the–week effects were somehow the result of a particular reserve management strategy by banks or reserve creation policy by the Fed, these would surely be related to the reserve maintenance period, which is a two–week rather than a one–week cycle. However, one accepts the null hypothesis that settlement Wednesdays and non–settlement Wednesdays on average have exactly the same level of cash holdings, as reported in the Υ_{jt} row of Table 5.

We accordingly take the view that although open market operations are responsible for the long–run trend in N_t, most of the variance of daily changes in N_t is due to fluctuations in the public's demand for cash. Banks simply pass daily variation in the public's demand for cash on to the Fed by asking to exchange deposits for Federal Reserve notes. Further support for the claim that disturbances to N_t can be treated as exogenous with respect to the other elements of the balance sheet is provided by the Granger–causality tests in Table 5. If fluctuations in N_t were a response to open market operations, one would expect lagged values of S_{t-j} to be helpful for forecasting N_t. In fact, they are of no help for forecasting, nor are lagged discount borrowing, Federal Reserve deposits, or the Treasury balance. Lagged values of the float F_{t-j} are statistically significant. However, the one lag that is individually significant is F_{t-4}, which enters with a negative sign. Such a pattern is highly implausible a priori, and we believe that the correct interpretation is that this is an example of rejecting the null hypothesis even though it is true, which of course should happen once in every twenty hypothesis tests. Indeed, when one groups F_{t-j} with the other lagged elements from the balance sheet, one readily accepts the joint null hypothesis that all twenty coefficients (including F_{t-4}) are zero.

We conclude that innovations in the estimated equation for N_t can be interpreted as shocks to the public's demand for cash and offer a second valid exogenous instrument for purposes of estimating structural models of Federal Reserve or bank reserve management behavior.

6. Float

Net Federal Reserve float (F_t) is displayed in Figure 3. On most days this magnitude is under a billion dollars, with storms and wire transfer errors occasionally generating large outliers. We partially control for outliers by introducing dummy variables for the two observations that are over ten billion dollars (March 15, 1993 and October 4, 1993) and for September 9, 1992, which, as described in Section 2, does not show up as an outlier in Figure 3 but was associated with some huge anomalies in the balance sheet. The point estimates for coefficients are similar with or without these dummy variables, though we regard the results that include the dummies as more reliable.

Our model of F_t is summarized in Table 7. Only one lag of float is useful for forecasting, and its coefficient is relatively small; there is not much serial correlation in this series.

We do find a strong day–of–the–week in the coefficient on ξ_{2t}; float tends to be half a billion dollars higher than usual on Tuesdays. Confirmation that this is something real rather than a statistical artifact is found in the significant positive coefficient on $H_{6,t-2}$— when there has been a three–day weekend ending on a Monday rather than the usual two–day weekend ending on a Sunday, the upsurge in float comes on Wednesday rather than Tuesday. In general, float is quite low the day after any holiday and rises subsequently, as captured by the coefficients on H_{3t} and H_{4t}. We interpret these patterns as resulting from lags in the check–clearing process and the extra volume the follows any day on which the check–clearing process is closed.

We also find that float is likely to be higher during the winter months (C_{3t}) and the post–Christmas season in particular ($X_{1t} + X_{6t}$), which we attribute to the fact that storms are more likely during those months. Float also tends to be slightly lower on the last day of the month (C_{3t}).

Lagged values of Federal Reserve notes (N_{t-j}) are useful for forecasting float. If banks asked the Fed for more currency yesterday ($N_{t-1} - N_{t-2}$ positive), then float tends to be higher today, with an additional billion in currency being associated with a $250 million increase in float. Such a pattern might result if people often ask for cash when they deposit a check.

We would again argue that the factors identified here as determinants of float— storms, volume of checks, and lags in the check–clearing process— are completely beyond the control of both banks and the Federal Reserve. Lagged

N_{t-j} was found to play a role, but we argued above that daily fluctuations in N_t are exogenous to banks and the Fed as well. We further find in Table 5 no contribution of the day of the maintenance period Υ_{jt} to float, and lagged values of any of the terms on the balance sheet other than F_{t-j} or N_{t-j} are no use in forecasting float.

We thus claim to have identified three sources of disturbance to items in the Federal Reserve's balance sheet that are largely beyond the control of both banks and the Fed itself. We associate the residuals in the equation for the Treasury balance U_t in Table 4 with unanticipated fiscal receipts and expenditures, the residuals in the equation for Federal Reserve notes N_t in Table 6 with changes in the public's demand for cash, and residuals in the equation for float F_t in Table 7 with temporary disturbances to the check–clearing system. As further evidence that these represent three separate exogenous sources of shocks, we document in the last row of Table 5 that the three residuals appear to be mutually uncorrelated; for example, one readily accepts the null hypothesis that contemporaneous values of N_t and F_t do not belong in the equation for U_t (p–value = 0.41).

If a disturbance to N_t has no effect on U_t or F_t, then, by the accounting identity, a \$1 billion increase in N_t must be associated with a \$1 billion increase in S_t or L_t, a \$1 billion decrease in D_t, or a combination of the three. If a disturbance to N_t is beyond the control of the Fed or banks, then the correlations one observes between N_t and S_t, L_t, or D_t must be interpreted as the response by banks and the Fed to disturbances in the demand for cash, rather than the other way around. In the subsequent sections we will therefore use the residuals of the equations for U_t, N_t, and F_t as instruments with which to identify a structural model of bank behavior.

7. Securities and other net assets

Adjustment of securities and other net assets, S_t, through open market opertions constitutes the Fed's primary policy tool. We do not attempt here to build a detailed structural model of the Fed's behavior. Instead we estimate a simple reduced–form equation relating S_t to the day of the maintenance period, lagged values of all the items in the balance sheet, and current shocks to the three equations for U_t, N_t, and F_t. The only coefficients of interest here are those on the last three variables, whose OLS estimates are as follows:

$$S_t = \mathbf{x}'_{St}\hat{\beta} + \underset{(0.051)}{0.474}\,\hat{\varepsilon}_{Ut} + \underset{(0.528)}{0.179}\,\hat{\varepsilon}_{Nt} + \underset{(0.136)}{0.045}\,\hat{\varepsilon}_{Ft}. \qquad (5)$$

Here $\mathbf{x}_{St}$ is a vector consisting of S_{t-j}, D_{t-j}, N_{t-j}, U_{t-j}, L_{t-j}, and Υ_{jt} for $j = 1,\ldots,10$, while $\hat{\varepsilon}_{Ut}$, $\hat{\varepsilon}_{Nt}$, and $\hat{\varepsilon}_{Ft}$ denote the fitted residuals from the OLS regressions reported in Tables 4, 6, and 7, respectively.

We have argued that these residuals represent shocks to specific structural equations; ε_{Ut} is a shock to Treasury receipts or expenditures, ε_{Nt} is a shock to the public's demand for cash, while ε_{Ft} is a shock to the payments clearing mechanism. We have further argued that these shocks are strictly exogenous with respect to monetary policy or the choices made by banks. We therefore interpret the strongly statistically significant coefficient on $\hat{\varepsilon}_{Ut}$ in equation (5) as the Federal Reserve's response to an exogenous shock to the Treasury balance. Specifically, the Fed is in consultation with the Treasury each day in an effort to forecast what the Treasury balance will be for that day. The Fed incorporates that forecast into the decision of whether and how much to add or drain reserves from the banking system that day. The positive coefficient on $\hat{\varepsilon}_{Ut}$ reflects the Fed's response to this advance information. Thus, if the Treasury balance experiences a \$1 billion shock on day t ($\varepsilon_{Ut} = 1$), the Fed typically adds \$474 million to its open market operations for that day, and thereby succeeds in partially insulating the banking system from the shock.

The positive coefficient on ε_{Nt} could likewise be interpreted as the Fed successfully anticipating part of the shock to cash demand for day t and managing to offset some of it through the open market operations. However, the coefficient on ε_{Nt} is estimated quite imprecisely, and is consistent with the claim that the Fed can not anticipate the shock at all, in which case the true coefficient would be zero, or that the Fed anticipates the shock perfectly, in which case the true coefficient could be unity.

By contrast, the small but positive coefficient on $\hat{\varepsilon}_{Ft}$ in equation (5) could not be interpreted as the Fed's response to an exogenous disturbance to the check clearing system, because it is of the wrong sign. Positive values for ε_{Ut} or ε_{Nt} take reserve deposits away from banks, whereas a positive value for ε_{Ft} adds reserve deposits to banks. However, the estimated coefficient on ε_{Ft} is quite close to zero and is reasonably well estimated; we would easily accept the null hypothesis that the Fed is completely unable to forecast disturbances to float, and can say with confidence that the Fed at most succeeds in neutralizing only a small part of any disturbance to float.

The residuals $\hat{\varepsilon}_{Ut}$, $\hat{\varepsilon}_{Nt}$, and $\hat{\varepsilon}_{Ft}$ in equation (5) are generated regressors, properties of which are discussed by Pagan (1984) and Murphy and Topel (1985). Specifically, for this example we know that (a) the coefficients reported in equation (5) give consistent estimates of what the coefficients would be on the true population variables ε_{Ut}, ε_{Nt}, or ε_{Ft}; (b) if the true coefficient is zero, then the standard errors reported in equation (5) give consistent estimates of the true standard errors. Thus a standard t–test of whether a coefficient in equation (5) is statistically significantly different from zero is perfectly valid. On the other hand, if the true coefficient is not zero, then the standard errors reported in equation (5) are incorrect.

One quick way to check for the importance of this issue is to replace the fitted residuals $\hat{\varepsilon}_{Ut}$, $\hat{\varepsilon}_{Nt}$, and $\hat{\varepsilon}_{Ft}$ with the ex–post values U_t, N_t, and F_t, and augment equation (5) to include all the explanatory variables used in the models for these three variables. Such a regression is free to replicate exactly the fit in (5), namely by choosing a coefficient on U_t equal to 0.474, a coefficient on U_{1t} equal to –0.474 times –2.30 (where –2.30 is the value of the coefficient relating U_t to U_{1t} in Table 4), and so on. If the goal is to estimate the response of S_t to the residuals in the equations for U_t, N_t, and F_t, such a regression is obviously highly inefficient, since it ends up estimating 43 free parameters for purposes of coming up with 3 particular coefficients. The regression is nevertheless of interest as a check on the validity of (5). The results of this regression are as follows:

$$S_t = \mathbf{x}'_{St}\hat{\boldsymbol{\gamma}} + \mathbf{z}'_t\hat{\boldsymbol{\delta}} + \underset{(0.049)}{0.471}\, U_t + \underset{(0.511)}{0.088}\, N_t + \underset{(0.132)}{0.040}\, F_t. \qquad (6)$$

Here $\mathbf{x}_{St}$ is the same vector of 60 lagged and deterministic variables appearing in (5) while $\mathbf{z}_t$ contains the 40 additional explanatory variables used in Tables 4, 6, and 7. The coefficients and standard errors in equation (6) are quite close to those in (5), and we will proceed on the assumption that the results in equation (5) are the appropriate ones to use.

To summarize, we find that a $1 billion shock to the Treasury balance is partially offset by a $474 million increase in the Fed's securities holdings. We earlier presented evidence that this shock is completely uncorrelated with the exogenous disturbances to either N_t or F_t. Thus from the accounting identity (1), a $1 billion shock to U_t must result in some combination of a decrease in banks' Federal Reserve deposits D_t and an increase in the loans banks take out from the discount window L_t, where the sum of these two effects must come to $526 million. Similarly, if we accept the 0.179 estimate in (5), a \$1 billion shock to N_t must also result in some combination of a decrease in D_t or an increase in L_t, where in this case $-\Delta D_t + \Delta L_t$ = \$821 million. Finally, the apparent zero coefficient on ε_{Ft} in (5) means that a \$1 billion shock to F_t must result in an increase in D_t or a decrease in L_t, with $\Delta D_t - \Delta L_t = 1$.

To know how a given shock gets apportioned between D_t and L_t, we need a model of banks' demand for excess reserves and willingness to borrow at the discount window, to which we turn next.

8. Discount window loans

This section develops a simple structural model of how banks react to exogenous shocks to their reserve position. Let X_t denote the supply of nonborrowed Federal Reserve Deposits that are available to banks on day t for purposes of meeting their current reserve requirement, where by "nonborrowed" we mean reserves that are not borrowed directly from the Fed itself at the discount window. Thus X_t is defined as

$$X_t = S_t + F_t - N_t - U_t. \tag{7}$$

Total Federal Reserve Deposits are the sum of X_t plus discount window loans:

$$D_t = X_t + L_t. \tag{8}$$

Banks face a biweekly reserve requirement, whereas our goal is to develop a model of daily reserve management. We define Q_t to be the average daily reserve requirement for the current two–week maintenance period. Note that Q_t is a step function, which is constant across different days of a given maintenance period but shifts up or down on the day that a new maintenance period begins. This constant Q_t has the property that if banks held this level of deposits every day of a given maintenance period they would just satisfy reserve requirements.$ We will refer to any deposits that banks hold above Q_t as the daily level of excess reserves E_t. Thus we have the accounting identity

$$Q_t + E_t = X_t + L_t. \tag{9}$$

We model required reserves Q_t and nonborrowed reserves X_t as exogenous to banks funds managers' decisions for day t.% In response to a decrease in X_t, the banking system as a whole must either borrow more from the Fed (increase L_t) or make do with a lower level of excess reserves (decrease E_t).

We assume that a representative bank perceives some benefits to holding positive excess reserves and some costs to letting E_t become negative. We model these through a general function $\alpha_t(E_t)$ characterizing the costs and benefits of excess reserves; note this is allowed to be a different function for different days t. The cost of letting E_t go negative on a typical day is that the bank must hold some extra reserves the following day. Hamilton

$Some comments are in order about how the series for Q_t was actually obtained. There are a variety of complications ignored in this study, such as the role of required clearing balances and applied versus nonapplied vault cash. Our goal was to obtain a series for Q_t that is consistent with the way other variables are measured in this study and reflects the key factors governing the demand for reserves. Our solution was to use the biweekly average series for seasonally unadjusted total reserves and required reserves from the FRED database maintained by the Federal Reserve Bank of St. Louis. We took the difference between these two series to obtain a biweekly average series for excess reserves $\bar{E}_t$. We then calculated a biweekly average of our own values for D_t over a given two-week period, counting typical Fridays triple, the day before one-day holidays double, and so on, to arrive at a step function $\bar{D}_t$ corresponding to average daily reserve deposits. We then subtracted average excess reserves $\bar{E}_t$ from $\bar{D}_t$ to arrive at the measure Q_t, which we interpret as average daily required Federal Reserve deposits.

%Our theoretical model assumes that banks know the value of average required reserves over the maintenance period Q_t at time t. In our empirical estimates, we will instrument for Q_t with variables known by banks at time t.

(1996) discussed some reasons why banks prefer not to do this and provided evidence that they are indeed willing to pay to avoid it. We assume that $\alpha_t' > 0$ (the more excess reserves, the better) and $\alpha_t'' < 0$ (there is a diminishing value of excess reserves or an increasing penalty for negative excess reserves).

Following Goodfriend, et. al. (1986) we also assume that banks act as if they faced a cost function for borrowing reserves from the Fed of the form $\beta_t(L_t)$. This function must be more than just the discount rate times the volume of loans, since typically the discount rate is less than the federal funds rate. If the only cost of borrowing from the Fed were the discount rate, then banks would want to borrow an infinite amount of reserves from the Fed at the discount rate, lend it all out at the federal funds rate, and pocket the difference. Banks obviously instead act as though they face nonpecuniary costs of borrowing, in the form of additional regulation, supervision, and inferior credit standing with other banks, if they place excessive reliance on the discount window. We assume that $\beta_t' > 0$ (borrowing more costs more) and $\beta_t'' > 0$ (the nonpecuniary marginal cost rises with the level of borrowing).

Condition (9) is an accounting identity that must hold for the aggregates. An individual bank, however, may contemplate lending any excess reserves it has to other banks on the Fed funds market, or borrowing there if it has a shortfall. Let A_t denote Fed funds lent by a representative bank, where $A_t < 0$ would indicate that this bank is borrowing Fed funds. If i_t denotes the federal funds rate (the interest rate on overnight loans of reserves between banks), then a representative bank is assumed to choose E_t, L_t, and A_t so as to maximize

$$\alpha_t(E_t) + A_t i_t - \beta_t(L_t)$$

subject to

$$Q_t + E_t + A_t = X_t + L_t. \tag{10}$$

An additional constraint is that banks can only borrow, not lend, at the discount window: $L_t \geq 0$. On a typical day, most banks do not go to the discount window, in which case the above problem would have the corner solution $L_t = 0$. In practice, however, aggregate L_t is never observed to be zero. On any given day, some banks are forced to borrow, even though most choose not to. A convenient way to model this is to generalize the constraint $L_t \geq 0$ to

$$L_t \geq \bar{L}_t. \tag{11}$$

Thus, our "representative" bank is forced to borrow an exogenous magnitude $\bar{L}_t$ at the discount window on day t, and will choose to borrow more than this only if it perceives the discount window to be attractive relative to the Fed funds market.

The first–order conditions for this maximization problem are

$$\alpha_t'(E_t) = i_t \tag{12}$$

$$\beta_t'(L_t) \geq i_t, \quad L_t \geq \bar{L}_t, \text{ and } (L_t - \bar{L}_t)(i_t - \beta_t'(L_t)) = 0. \tag{13}$$

Equation (12) states that the opportunity cost of Fed funds lent (i_t) must equal the marginal benefit of holding more excess reserves ($\alpha_t'(E_t)$). Equation (13) states that either the marginal cost of discount borrowing ($\beta_t'(L_t)$) exceeds the cost of Fed funds borrowing, in which case the bank would opt for the minimal level of discount borrowing $\bar{L}_t$, or else the bank chooses to borrow at the discount window ($L_t > \bar{L}_t$) up to the point where the marginal cost of discount borrowing equals the marginal cost of borrowing on the Fed funds market.

We assume that the benefit function for excess reserves on day t ($\alpha_t(\cdot)$) and the cost of borrowed reserves function ($\beta_t(\cdot)$) are quadratic,

$$\alpha_t(E_t) = h_t + c_tE_t - (1/2)a_tE_t^2 + \varepsilon_{\alpha t}E_t$$
$$\beta_t(L_t) = k_t + d_tL_t + (1/2)b_tL_t^2 + \varepsilon_{\beta t}L_t,$$

where $\varepsilon_{\alpha t}$ and $\varepsilon_{\beta t}$ represent shocks to marginal costs or benefits that are seen by banks but not the econometrician, and h_t and k_t are arbitrary constants for the cost functions. The first–order condition (12) then becomes

$$i_t = c_t - a_tE_t + \varepsilon_{\alpha t} \tag{14}$$

while (13) implies

$$L_t = \max\{\bar{L}_t, L_t^*\} \tag{15}$$

where L_t^* satisfies

$$i_t = d_t + b_tL_t^* + \varepsilon_{\beta t}. \tag{16}$$

Equations (10), (14), (15), and (16) describe the values of A_t, E_t, L_t, and L_t^* chosen by a representative bank as functions of Q_t, X_t, i_t, and $\bar{L}_t$.

If one bank lends Fed funds, another must borrow, so that equilibrium requires that the representative bank is induced to choose $A_t = 0$. The variable that adjusts to ensure this equilibrium condition is i_t. Setting $A_t = 0$, equations (10), (14), (15) and (16) then describe the equilibrium values of E_t, i_t, L_t, and L_t^* as functions of Q_t, X_t and $\bar{L}_t$. These equations can be explicitly solved as

$$L_t^* = \frac{c_t - d_t}{b_t + a_t} - \frac{a_t}{b_t + a_t}(X_t - Q_t) + \frac{\varepsilon_{\alpha t} - \varepsilon_{\beta t}}{b_t + a_t} \tag{17}$$

$$L_t = \max\{L_t^*, \bar{L}_t\} \tag{18}$$

$$E_t = X_t + L_t - Q_t \tag{19}$$

$$i_t = c_t - a_tE_t + \varepsilon_{\alpha t}. \tag{20}$$

Significant levels of discount window borrowing take place only on settlement Wednesday or the last day of a quarter (see Figure 4). This could be explained in terms of the above equations if banks regard the marginal cost of discount borrowing b_t to be much higher on nonsettlement days, that is, the

Fed subjects a bank's request to borrow to less scrutiny when that request comes on a settlement day or at the end of a quarter. For this reason we consider the following specification:

$$b_t = \beta_1(1 - C_{6t}) + \beta_2 C_{6t} \tag{21}$$

$$d_t = \delta_1(1 - C_{6t}) + \delta_2 C_{6t}, \tag{22}$$

where $C_{6t} = 1$ if day t is the last day of a maintenance period or the last day of a quarter. Equations (21) and (22) imply that the marginal cost of discount borrowing is $\delta_2 + \beta_2 L_t + \varepsilon_{\beta t}$ if t is the last day of a maintenance period or the last day of a quarter, whereas the marginal cost is $\delta_1 + \beta_1 L_t + \varepsilon_{\beta t}$ on other days. If we similarly specify

$$a_t = \alpha_1(1 - C_{6t}) + \alpha_2 C_{6t} \tag{23}$$

$$c_t = \gamma_1(1 - C_{6t}) + \gamma_2 C_{6t} \tag{24}$$

and assume a second–order autoregressive structure for $(\varepsilon_{\alpha t} - \varepsilon_{\beta t})$, then we obtain the following expression whenever $L_t = L_t^*$:&

$$\begin{aligned} L_t = {} & \frac{\gamma_1 - \delta_1}{\beta_1 + \alpha_1}(1 - C_{6t}) + \frac{\gamma_2 - \delta_2}{\beta_2 + \alpha_2} C_{6t} - \frac{\alpha_1}{\beta_1 + \alpha_1}(1 - C_{6t})(X_t - Q_t) \\ & - \frac{\alpha_2}{\beta_2 + \alpha_2} C_{6t}(X_t - Q_t) + \sum_{s=1}^{2} [\phi_s C_{6t} L_{t-s} + \eta_s (1 - C_{6t}) L_{t-s}] \\ & + \sum_{s=1}^{2} [\psi_s C_{6,t-s} L_{t-s} + \omega_s (1 - C_{6,t-s}) L_{t-s}] + v_t . \end{aligned} \tag{25}$$

Equation (25) is a reduced form from the point of view of the economic model— the magnitude $X_t - Q_t$ is taken as exogenous with respect to banks' decisions. Even so, equation (25) is not a reduced form in terms of its econometric properties, because the error v_t is correlated with the regressor $(X_t - Q_t)$. The reason is that the Federal Reserve chooses the quantity of

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&Note that since each pair of regressors within square brackets in (25) sum to L_{t-s}, two of the eight regressors within square brackets in (25) are redundant.

nonborrowed reserves X_t so as to target the federal funds rate. To do this, it will actively adjust X_t in response to perceived shocks to banks' reserve needs; in other words, X_t will respond to $\varepsilon_{\beta t}$ and $\varepsilon_{\alpha t}$. Fortunately, the earlier analysis has suggested three valid instruments for $(X_t - Q_t)$, namely, ε_{Ut}, ε_{Nt}, and ε_{Ft}, or the residuals for the equations for the Treasury balance (U_t), Federal Reserve notes outstanding (N_t), and net float (F_t), respectively. Multiplying these residuals by C_{6t} or $(1 - C_{6t})$ yields a total of six variables which can instrument for $C_{6t}(X_t - Q_t)$ and $(1 - C_{6t})(X_t - Q_t)$ in (25). Two–stage least squares estimation of (25) (using the other variables in (25) as additional instruments) produced the following estimates:[7]

$$
\begin{aligned}
L_t = {} & \underset{(0.018)}{0.052} + \underset{(0.067)}{0.744}\, C_{6t} - \underset{(0.011)}{0.008}\,(1 - C_{6t})(X_t - Q_t) \\
& - \underset{(0.015)}{0.158}\, C_{6t}(X_t - Q_t) + \underset{(0.089)}{0.552}\,(1 - C_{6,t-1})L_{t-1} \\
& + \underset{(0.084)}{0.222}\,(1 - C_{6,t-2})L_{t-2} + \underset{(0.306)}{1.142}\, C_{6t}L_{t-1} - \underset{(0.382)}{1.997}\, C_{6t}L_{t-2} \\
& + \underset{(0.044)}{0.001}\, C_{6,t-1}L_{t-1} + \underset{(0.160)}{0.511}\, C_{6,t-1}L_{t-2}. \qquad (26)
\end{aligned}
$$

The near–zero coefficient on $(1 - C_{6t})(X_t - Q_t)$ indicates that a shock to the banking system's level of free reserves $(X_t - Q_t)$ has no effect on discount window borrowing unless it occurs on the last day of the quarter or the last day of a maintenance period. We interpret this as meaning that on a typical day, when $C_{6t} = 0$, the marginal cost of discount window borrowing b_t is sufficiently high that one always observes $L_t = \bar{L}_t$, the minimal level of borrowing, for those days. Recalling the accounting identity (9), this means that any shock to free reserves on a day when $C_{6t} = 0$ is entirely absorbed by an increase or decrease in that day's excess reserves E_t. By contrast, on

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[7]The instruments used for estimation of (26) were a constant and $\hat{\varepsilon}_{Ut}$, $C_{6t}\hat{\varepsilon}_{Ut}$, $\hat{\varepsilon}_{Nt}$, $c_{6t}\hat{\varepsilon}_{Nt}$, $\hat{\varepsilon}_{Ft}$, $C_{6t}\hat{\varepsilon}_{Ft}$, C_{6t}, $(1 - C_{6,t-1})L_{t-1}$, $(1 - C_{6,t-2})L_{t-2}$, $C_{6t}L_{t-1}$, $C_{6t}L_{t-2}$, $C_{6,t-1}L_{t-1}$, $C_{6,t-1}L_{t-2}$.

settlement days or the end of a quarter, when $C_{6t} = 1$, our structural estimate is that

$$\frac{\hat{\alpha}_2}{\hat{\beta}_2 + \hat{\alpha}_2} = 0.158,$$

that is, banks borrow an additional $158 million at the discount window on these days in response to a $1 billion negative shock to free reserves, and reduce their excess reserves by $842 million.

Next, we investigate the first–order condition (14) directly. Our approach here is to start by trying to replicate the model of the federal funds rate used in Hamilton (1996), which describes Δi_t in terms of assorted calendar factors:

$$\Delta i_t = \sum_{j=1}^{10} \eta_j \Upsilon_{jt} + \phi \Upsilon_{1t}(i_{t-1} - i_{t-3}) + \beta_1 H_{5t} + \beta_2 H_{6t} + \beta_3 H_{5,t-1} + \beta_4 H_{6,t-1} + v_t. \qquad (27)$$

One important difference is that while Hamilton (1996) included a detailed allowance for outliers and ARCH features of v_t, here we simply estimate (27) by OLS. In both studies i_t represents the effective federal funds rate, which is a volume–weighted average of the rate at which all the brokered trades for day t take place. In Hamilton (1996), the sample consisted of 1700 observations between 1984 and 1990, whereas the current study uses 756 observations from 1992 to 1994.

Table 8 compares the original maximum likelihood estimates reported in Hamilton (1996) with OLS estimates for the new sample. The standard errors are significantly bigger with a shorter sample period and no correction for outliers. Nevertheless, all of the main patterns found in the earlier study are confirmed in the new data. The federal funds rate tends to fall on Fridays (days 2 and 7) and Tuesdays (days 4 and 9) and rise on Mondays (days 3 and 8). A huge surge in the funds rate often occurs on settlement Wednesday (day 10), with about 3/4 of any move on the last two days of a maintenance period being reversed at the beginning of a new period, as reflected in the coefficient on $\Upsilon_{1t}(i_{t-1} - i_{t-3})$.

We then added $C_{6t}E_t$ and $(1 - C_{6t})E_t$ as explanatory variables in (27) and estimated the resulting equation by two–stage least squares,

$$\Delta i_t = \mathbf{x}'_{it}\hat{\beta} - \underset{(0.0108)}{0.0257}(1 - C_{6t})E_t - \underset{(0.0170)}{0.0784} C_{6t}E_t, \qquad (28)$$

where $\mathbf{x}_{it}$ denotes the vector of explanatory variables appearing in (27). The instruments used for estimation of (28) were $\mathbf{x}_{it}$ along with $\hat{\varepsilon}_{jt}$ and $C_{6t}\hat{\varepsilon}_{jt}$ for j = U, N, and F. We interpret 0.0257 as an estimate of α_1, or the marginal benefit that banks perceive to holding excess reserves on a day that is not a settlement day or the end of a quarter, whereas 0.0784 is an estimate of α_2, the marginal benefit of excess reserves on settlement day or the end of a quarter. Note both estimates are of the expected sign and statistically significant.

9. Interpretation and corroboration.

Our structural model of banks' willingness to borrow at the discount window led to the following conclusions. Unless it is the end of a maintenance period or the last day of a quarter, a typical bank does not go to the discount window in response to an exogenous shock to nonborrowed reserves. Instead it goes to the federal funds market, and the banking system as a whole makes do with reserves correspondingly below the average daily requirement. However, banks are reluctant to do this, and the equilibrium federal funds rate is bid up as a consequence. Equilibrium requires that the funds rate must go up by the marginal benefit of excess reserves α_1 times the amount of lost reserves, or a 2.6–basis–point increase in the federal funds rate for every \$1 billion in lost reserves. We therefore conclude that a temporary \$1 billion open market sale on such days would raise the federal funds rate by 2.6 basis points. Note this replicates the estimate in Hamilton (1997) that a \$1 billion open market sale on these days would produce a 1– to 3–basis–point increase in the federal funds rate.

On the other hand, on settlement days or the end of a quarter, banks have a higher perceived marginal benefit of excess reserves, and will make up some of the shortfall with additional discount window borrowing. The equilibrium condition (14) still must hold, however— the federal funds rate should rise by the marginal benefit of excess reserves α_2 times the amount by which banks are forced to reduce their reserve holdings. In response to a \$1 billion open market sale on settlement day or the end of a quarter, our estimates suggest that banks would borrow an additional \$158 million at the discount window so that their excess reserves fall by \$842 million. The predicted effect of a \$1 billion open market sale on these days is therefore an increase in the federal funds rate of (0.842)(0.0784) = 0.0660 or an increase of 6.6 basis points. This is substantially smaller than the estimate of 22.7 basis points obtained in Hamilton (1997).(

The analysis in Sections 4–7 of the sources of shocks to banks' nonborrowed reserves suggests a number of natural experiments with which to assess these predictions. For example, disturbances to the Treasury balance are one source of exogenous shock. We saw in equation (5) that nearly half of a typical disturbance to the Treasury's balance is neutralized with an offsetting open market operation, meaning that on average a shock to ε_{Ut} will reduce nonborrowed reserves by \$526 million. Given the estimate α_1 = 0.0257,

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(Hamilton (1997) estimated that a \$440 million open market sale on settlement day would raise the federal funds rate by 10 basis points, meaning a \$1 billion sale would produce a move of 10 ÷ 0.44 = 22.7 basis points.

this means that a \$1 billion shock to ε_{Ut} when $C_{6t} = 0$ should result in an increase in the federal funds rate of $(-0.526)(-0.0257) = 0.014$, or an increase of 1.4 basis points. The same value for ε_{Ut} on settlement day or the last day of a quarter (when $C_{6t} = 1$) would be predicted to result in an increase in the federal funds rate of $(-0.526)(-0.0660) = 0.035$. Similarly, according to the estimate in equation (5), a \$1 billion shock to the public's cash demand ε_{Nt} typically reduces nonborrowed reserves by \$821 million. When $C_{6t} = 0$ this should mean an increase in the federal funds rate of $(-0.821)(-0.0257) = 0.021$, whereas when $C_{6t} = 1$ the federal funds rate would be predicted to rise by $(-0.821)(-0.0660) = 0.054$. Finally, given that the Fed seems completely unable to predict disturbances to float, a \$1 billion shock to ε_{Ft} increases banks' nonborrowed reserves by \$1 billion and so should reduce the federal funds rate by 0.0257 when $C_{6t} = 0$ and by 0.0660 when $C_{6t} = 1$. These predictions are summarized in Table 9.

This experiment— exogenously disturb ε_{Ut}, ε_{Nt}, or ε_{Ft} and see what happens to i_t— is in fact run every day. Recalling that the three disturbances appear to be uncorrelated, we can read the results of this experiment directly off an OLS regression of the change in the federal funds rate on the exogenous disturbances. When these variables are added to the interest rate regression (27), the OLS estimates are

$$\begin{aligned}\Delta i_t = \mathbf{x}_{it}'\hat{\boldsymbol{\gamma}} &+ \underset{(0.0054)}{0.0120}\,(1 - C_{6t})\hat{\varepsilon}_{Ut} + \underset{(0.0103)}{0.0385}\,C_{6t}\hat{\varepsilon}_{Ut} \\ &- \underset{(0.0522)}{0.0155}\,(1 - C_{6t})\hat{\varepsilon}_{Nt} + \underset{(0.1552)}{0.4247}\,C_{6t}\hat{\varepsilon}_{Nt} \\ &- \underset{(0.0135)}{0.0168}\,(1 - C_{6t})\hat{\varepsilon}_{Ft} - \underset{(0.0359)}{0.1222}\,C_{6t}\hat{\varepsilon}_{Ft}. \qquad (29)\end{aligned}$$

The six coefficients reported in (29) represent six separate natural experiments with which to assess whether there is a liquidity effect, that is, whether taking reserves away from banks causes the federal funds rate to go up. Positive values of ε_{Ut} and ε_{Nt} drain reserves whereas a positive value for ε_{Ft} adds reserves. Note that all but one of the coefficients have the predicted sign; the one exception is the coefficient on $(1 - C_{6t})\hat{\varepsilon}_{Nt}$, and this is not statistically significant. Of the other five coefficients, four are statistically significantly different from zero— there are little grounds for disputing that draining reserves raises the federal funds rate, particularly on settlement day.

Table 9 compares the estimated effects of these exogenous disturbances with those predicted on the basis of the theoretical calculations presented above equation (29). By far the most precisely estimated coefficients are those on disturbances to the Treasury balance, and here the correspondence between the predicted magnitude and the OLS estimate is quite remarkable. The coefficients on ε_{Ft} are also fairly close to their predicted values. The one significant difference between the predicted and actual experimental outcome is the coefficient on $C_{6t}\hat{\varepsilon}_{Nt}$, which is an order of magnitude bigger than the theoretical calculations would predict. Even though this coefficient is estimated very imprecisely, it is fair to say that the size of the liquidity effect captured by this coefficient is too big to be explained by the theoretical framework presented in Section 8. Given the substantial corroboration of this framework from the variety of other sources reported here, we are inclined to give less credibility to the large coefficient on $C_{6t}\hat{\varepsilon}_{Nt}$. Possibly we have omitted some of the factors influencing the demand for cash that may exert an independent influence on the federal funds rate, possibly there is some deliberate manipulation of cash balances by banks on settlement day or the end of a quarter that undermines the assumed exogeneity of this magnitude, or possibly there is a problem of limited data and nonlinearities, if some big cash demand disturbances happened to occur on days of a very tight Fed funds market.

This coefficient notwithstanding, it is surely correct to conclude that when the Fed drains reserves from banks, the effect is to raise the federal funds rate. The mechanism for this effect is quite clear. On days other than settlement Wednesday or the end of a quarter, banks simply get by with a lower level of reserves. They are reluctant to do so, however, and accordingly will bid up the federal funds rate. The funds rate rises to a point such that the new lower level of reserves is an equilibrium, namely to a point such that banks' perceived marginal benefit of holding excess reserves is equal to the federal funds rate.

The marginal benefit of excess reserves is much higher on settlement days or the end of a quarter. If the Fed drains reserves on these days, banks will replace some of the lost reserves with discount window borrowing and the movement in the federal funds rate needed to restore equilibrium will be substantially bigger.

The specific estimates developed here imply that a $1 billion open market sale would drive up the federal funds rate by 6.6 basis points on settlement day or the end of a quarter and by 2.6 basis points on other days.

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Table 1

Balance sheet of the Federal Reserve System, March 4, 1992
(billions of dollars)

ASSETS		
(S) **Securities and other net Federal Reserve assets**		**319.158**
U.S. Treasury securities		
bills bought outright [11]	129.798	
notes bought outright [12]	102.835	
bonds bought outright [13]	32.043	
held under repurchase agreements [14]	2.016	
federal agency obligations		
bought outright [7]	5.960	
held under repurchase agreements [8]	0.111	
assets denominated in other currencies [18]	26.002	
gold certificate account [1]	11.058	
special drawing rights certificate account [2]	10.018	
coin [3]	0.623	
bank premises [17]	1.000	
all other Federal Reserve assets [19]	5.003	
other Federal Reserve liabilities –[28]	(2.224)	
capital paid in –[30]	(2.734)	
surplus –[31]	(2.342)	
other capital accounts –[32]	(0.009)	
(L) **Discount window loans**		**0.044**
to depository institutions [4]	0.044	
other [5]	0	
acceptances held under repurchase agreements [6]	0	
(F) **Net float**		**0.087**
items in process of collection [16]	6.440	
deferred credit items –[27]	(5.827)	
Federal Reserve deposits held by foreign official accounts and other [23] + [24] –[22]	(0.526)	
LIABILITIES		
(N) **Federal Reserve notes** [21]		**282.498**
(D) **Federal Reserve deposits held by depository institutions** [23]		**30.478**
(U) **Federal Reserve deposits held by the U.S. Treasury** [24]		**6.313**

Notes to Table 1.

Numbers in brackets refer to the series number in Table 1.18 of the Federal Reserve Bulletin, June 1992. Entries with parentheses are subtracted to calculate subtotal items.

Table 2

Replication of Estimation of Treasury Balance Model from Hamilton (1997)

explanatory variable	previous estimates coefficient	previous estimates (standard error)	new estimates coefficient	new estimates (standard error)
constant	2.35	(0.22)	3.52	(0.43)
U_{t-1}	0.54	(0.04)	0.41	(0.08)
U_{t-2}	–0.05	(0.03)	–0.05	(0.04)
U_{t-3}	0.08	(0.02)	0.02	(0.03)
U_{1t}	–3.64	(0.71)	–2.07	(0.71)
$U_{1t}U_{t-1}$	0.46	(0.07)	0.38	(0.09)
U_{2t}	8.92	(1.31)	3.10	(1.78)
$U_{2t}U_{t-1}$	–0.99	(0.08)	–0.50	(0.10)
U_{3t}	0.67	(0.24)	1.62	(0.25)
γ_{4t}	–0.46	(0.13)	–0.89	(0.25)
ξ_{5t}	–0.36	(0.09)	–0.39	(0.17)
$I[C_{1t}=3]$	–0.32	(0.16)	–0.37	(0.33)
$I[C_{1t}=4]$	0.61	(0.17)	0.13	(0.32)
C_{2t}	0.94	(0.37)	1.79	(0.32)

Table 3

Dummy variable notation

Calendar indicators

- $\gamma_{jt} = 1$ if day t falls in month j (j = 1,...,12)
- $\xi_{jt} = 1$ if day t falls on day j of the week (j = 1,...,5)
- $\Upsilon_{jt} = 1$ if day t falls on day j of a reserve maintenance period, (j = 1,..,10)
- $C_{1t} = j$ if day t is the jth business day of the current month
- $C_{2t} = 1$ if day t is the last business day of the current month
- $C_{3t} = 1$ if day t falls in winter ($C_{3t} = \gamma_{12,t} + \gamma_{1t} + \gamma_{2t} + \gamma_{3t}$)
- $C_{4t} = j$ if day t falls in year j (j = 92, 93, 94)
- $C_{5t} = 1$ if day t is last day of maintenance period ($C_{5t} = \Upsilon_{10,t} + \Upsilon_{9t} I[H_{5t} + H_{6t} = 1]$)
- $C_{6t} = 1$ if day t is last day of maintenance period or the last day of a quarter

Tax indicators

- $U_{1t} = 1$ if previous day exceeded threshold ($U_{1t} = I[U_{t-1} > 8]$)
- $U_{2t} = 1$ if t is first day of month and previous day exceeded threshold ($U_{2t} = U_{1t} I[C_{1t} = 1]$)
- $U_{3t} = 1$ for major tax collection periods identified in Hamilton (1997); t comes after the first Monday following the 15th and falls in January, April, June, or September
- $U_{4t} = 1$ for broader definition of major tax collection periods; t comes after the first Monday following the 15th and falls in January, April, June, September, or December
- $U_{5t} = 1$ for the first day of a tax collection period identified by U_{4t}

Holiday indicators

- $H_{1t} = 1$ if day t is 1, 2, 3, 4, or 5 business days before one of the holidays outside the Christmas season (President's Day, Memorial Day, July 4, Labor Day, Columbus Day, Veteran's Day)
- $H_{2t} = j$ if $H_{1t} = 1$ and it will be j business days until the holiday (j = 1,2,...,5)
- $H_{3t} = 1$ if day t is 1, 2, 3, 4, or 5 business days after one of the holidays outside the Christmas season

H_{4t} = j if H_{3t} = 1 and it has been j business days since the holiday (j = 1,2,...,5)
H_{5t} = 1 if day t comes before any 1–day holiday
H_{6t} = 1 if day t comes before any 3–day holiday

Christmas indicators

X_{1t} = 1 if day t is 1 or 2 business days before Christmas or 1 to 3 days after
X_{2t} = 1 if day t is 3 to 15 business days before Christmas
X_{3t} = j if X_{2t} = 1 and it will be j business days until Christmas (j = 3,4,...,15)
X_{4t} = 1 if day t is 1 to 5 business days before Thanksgiving
X_{5t} = j if X_{4t} = 1 and it will be j business days until Thanksgiving (j = 1,2,...,5)
X_{6t} = 1 if day t is 4 to 25 business days after Christmas
X_{7t} = j if X_{6t} = 1 and it has been j business days since Christmas (j = 4,5,...,25)

Balance sheet anomaly indicators

F_{1t} = 1 for t = September 9, 1992
F_{2t} = 1 for t = March 15, 1993
F_{3t} = 1 for t = October 4, 1993

Notes to Table 2.

All variables take on the value zero except under the conditions indicated in the table, and I[·] is the indicator function; I[X] = 1 if condition X is true and is zero otherwise.

Table 4

Updated Model of the Treasury Balance

explanatory variable	coefficient	(standard error)
U_{t-1}	0.37	(0.07)
U_{t-2}	–0.02	(0.04)
U_{t-3}	–0.05	(0.04)
U_{t-4}	0.08	(0.03)
U_{1t}	–2.30	(0.70)
$U_{1t}U_{t-1}$	0.43	(0.08)
U_{2t}	2.90	(1.72)
$U_{2t}U_{t-1}$	–0.51	(0.10)
U_{4t}	1.02	(0.23)
U_{5t}	3.01	(0.51)
γ_{4t}	–0.74	(0.24)
ξ_{1t}	3.54	(0.44)
ξ_{2t}	3.74	(0.43)
ξ_{3t}	3.39	(0.44)
ξ_{4t}	3.02	(0.44)
ξ_{5t}	3.14	(0.44)
C_{2t}	1.89	(0.31)

Table 5

Specification Tests for Models of U_t, N_t, and F_t

	Model of U_t		Model of N_t		Model of F_t	
omitted variables	d.f.	p–value	d.f.	p–value	d.f.	p–value
t	1	0.24	—	—	1	0.96
γ_{jt} j=1,...,12	10	0.15	10	0.02*	10	0.75
ξ_{jt} j=1,...,5	—	—	—	—	3	0.66
Υ_{jt} j=1,...,10	5	0.51	5	0.23	8	0.68
C_{1t}, C_{1t}^2	2	0.99	—	—	2	0.91
C_{2t}, C_{3t}	1	0.28	2	0.19	—	—
$\xi_{jt} I[C_{4t}=92]$ j=1,...,5	5	0.68	5	0.08	5	0.38
$\xi_{jt} I[C_{4t}=93]$ j=1,...,5	5	0.36	5	0.08	5	0.22
$\xi_{jt} I[C_{4t}=94]$ j=1,...,5	5	0.28	—	—	5	0.14
$U_{1t}, U_{1t}U_{t-1}, U_{2t}, U_{2t}U_{t-1}, U_{3t}, U_{4t}, U_{5t}$	1	0.06	7	0.40	7	0.24
H_{jt} j=1,...,4	4	0.70	2	0.05	2	0.10
$H_{5t}, H_{5,t-1}, H_{6t}, H_{6,t-1}$	4	0.70	2	0.42	4	0.58
$I[C_{4t}=92] \times \{H_{5t}, H_{5,t-1}, H_{6t}, H_{6,t-1}\}$	4	0.99	4	0.65	4	0.69
$I[C_{4t}=93] \times \{H_{5t}, H_{5,t-1}, H_{6t}, H_{6,t-1}\}$	4	0.67	4	0.38	4	0.78
$I[C_{4t}=94] \times \{H_{5t}, H_{5,t-1}, H_{6t}, H_{6,t-1}\}$	4	0.92	—	—	4	0.25

Table 5 (continued)

omitted variables	Model of U_t d.f.	Model of U_t p–value	Model of N_t d.f.	Model of N_t p–value	Model of F_t d.f.	Model of F_t p–value
X_{jt} j=1,...,7	7	0.20	——	——	6	0.67
$F_{jt}, F_{j,t-1}$ j=1,2,3	6	0.93	6	0.62	——	——
S_{t-j} j=1,...,5	5	0.33	5	0.95	5	0.66
L_{t-j} j=1,...,5	5	0.81	5	0.77	5	0.71
F_{t-j} j=1,...,5	5	0.14	5	0.02*	4	0.60
N_{t-j} j=1,...,5	5	0.14	——	——	——	——
D_{t-j} j=1,...,5	5	0.16	5	0.70	5	0.33
U_{t-j} j=1,...,5	1	0.85	5	0.27	5	0.20
$S_{t-j}, L_{t-j}, F_{t-j}, N_{t-j}, D_{t-j}, U_{t-j}$	21	0.28	20	0.35	19	0.39
U_t, N_t, F_t	2	0.41	2	0.53	2	1.00

Notes to Table 5.

"d.f." stands for degrees of freedom, or the number of restrictions being tested. The null hypothesis in each case is that none of the variables in the list at the left of the row (with the exception of any of these variables that may already be included in the particular model) belong in the regression explaining the variable labeled in each column.

"p–value" is the probability of having generated as large an F statistic as was calculated if the indicated null hypothesis were true

"——" means that all of the listed variables are already included in that model.

"*" indicates statistically significant at the 5% level.

Table 6

Model of Federal Reserve Notes

explanatory variable	coefficient	(standard error)
t	0.00251	(0.00052)
N_{t-1}	1.21	(0.04)
N_{t-2}	–0.09	(0.05)
N_{t-3}	–0.05	(0.05)
N_{t-4}	–0.03	(0.05)
N_{t-5}	0.01	(0.04)
N_{t-6}	–0.20	(0.04)
N_{t-7}	0.05	(0.04)
N_{t-8}	0.09	(0.04)
N_{t-9}	–0.03	(0.04)
N_{t-10}	0.03	(0.02)
$\gamma_{12,t}$	–0.15	(0.06)
ξ_{1t}	5.96	(1.13)
ξ_{2t}	6.06	(1.13)
ξ_{3t}	5.74	(1.14)
ξ_{4t}	5.92	(1.13)
ξ_{5t}	5.34	(1.14)
$\xi_{1t}I[C_{4t}=94]$	0.53	(0.05)

Table 6 (continued)

explanatory variable	coefficient	(standard error)
$\xi_{2t} I[C_{4t}=94]$	0.49	(0.05)
$\xi_{3t} I[C_{4t}=94]$	0.22	(0.05)
$\xi_{4t} I[C_{4t}=94]$	–0.59	(0.05)
$\xi_{5t} I[C_{5t}=94]$	–0.18	(0.05)
C_{1t}	–0.0507	(0.0057)
C_{1t}^2	0.0021	(0.0002)
H_{1t}	0.54	(0.05)
H_{2t}	–0.089	(0.015)
$H_{5t} I[C_{4t}=94]$	–0.55	(0.20)
$H_{5,t-1} I[C_{4t}=94]$	0.27	(0.19)
$H_{6t} I[C_{4t}=94]$	0.51	(0.07)
$H_{6,t-1} I[C_{4t}=94]$	–0.25	(0.08)
X_{1t}	0.47	(0.08)
X_{2t}	0.90	(0.10)
X_{3t}	–0.046	(0.008)
X_{4t}	0.80	(0.13)
X_{5t}	–0.11	(0.04)
X_{6t}	–0.36	(0.07)
X_{7t}	0.010	(0.004)

Table 7

Model of Net Float

explanatory variable	coefficient	(standard error)
constant	–0.65	(0.29)
F_{t-1}	0.25	(0.03)
N_{t-1}	0.25	(0.07)
N_{t-2}	–0.38	(0.14)
N_{t-3}	0.14	(0.13)
N_{t-4}	0.12	(0.10)
N_{t-5}	–0.13	(0.06)
ξ_{2t}	0.46	(0.09)
C_{2t}	–0.30	(0.12)
C_{3t}	0.22	(0.06)
H_{3t}	–0.84	(0.21)
H_{4t}	0.21	(0.06)
$H_{6,t-2}$	2.01	(0.18)
$X_{1t} + X_{6t}$	0.49	(0.10)
F_{1t}	–2.95	(0.69)
$F_{1,t-1}$	1.07	(0.68)
F_{2t}	10.11	(0.68)
$F_{2,t-1}$	–1.56	(0.76)

Table 7 (continued)

explanatory variable	coefficient	(standard error)
F_{3t}	10.33	(0.68)
$F_{3,t-1}$	–2.44	(0.76)

Table 8

Replication of Estimation of Federal Funds Rate Equation from Hamilton (1996)

explanatory variable	previous estimates coefficient	previous estimates (standard error)	new estimates coefficient	new estimates (standard error)
Υ_{1t}	0.018	(0.008)	–0.020	(0.029)
Υ_{2t}	–0.040	(0.002)	–0.119	(0.028)
Υ_{3t}	0.041	(0.007)	0.100	(0.028)
Υ_{4t}	–0.036	(0.006)	–0.086	(0.027)
Υ_{5t}	–0.036	(0.006)	–0.031	(0.026)
Υ_{6t}	0.008	(0.006)	0.023	(0.026)
Υ_{7t}	–0.034	(0.006)	–0.072	(0.028)
Υ_{8t}	0.057	(0.008)	0.157	(0.029)
Υ_{9t}	–0.045	(0.009)	–0.136	(0.028)
$\Upsilon_{10,t}$	0.139	(0.023)	0.244	(0.027)
$\Upsilon_{1t}(i_{t-1}-i_{t-3})$	–0.811	(0.021)	–0.700	(0.062)
H_{5t}	–0.028	(0.020)	0.350	(0.107)
$H_{5,t-1}$	0.023	(0.020)	–0.121	(0.107)
H_{6t}	–0.031	(0.011)	–0.155	(0.053)
$H_{6,t-1}$	0.171	(0.017)	0.498	(0.054)

Table 9

Theoretical predictions and OLS estimates of effects of exogenous disturbances on the federal funds rate

	Treasury balance (U_t)	Federal Reserve notes (N_t)	Net float (F_t)
Presumed effect on nonborrowed reserves	–0.526	–0.821	1.000
Effect on i_t when $C_{6t} = 0$			
predicted effect	0.014	0.021	–0.026
estimated effect	0.012	–0.015	–0.017
(standard error)	(0.005)	(0.052)	(0.014)
p–value for estimated = 0	0.028*	0.767	0.215
p–value for estimated = predicted	0.706	0.485	0.494
Effect on i_t when $C_{6t} = 1$			
predicted effect	0.035	0.054	–0.066
estimated effect	0.039	0.425	–0.122
(standard error)	(0.010)	(0.155)	(0.036)
p–value for estimated = 0	0.000**	0.006**	0.001**
p–value for estimated = predicted	0.964	0.017*	0.118