Orthogonalized VARs

A. Recursively orthogonalized VAR
B. Variance decomposition
C. Historical decomposition
D. Structural interpretation
E. Generalized IRFs

A. Recursively orthogonalized VARs

Nonorthogonal IRF:

$$\underset{(n\times n)}{\boldsymbol{\Psi}_s} = \frac{\partial E(\mathbf{y}_{t+s}|\mathbf{y}_t,\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial \mathbf{y}_t'}$$

Column 1 $= \dfrac{\partial E(\mathbf{y}_{t+s}|y_{1t},y_{2t},\ldots,y_{nt},\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial y_{1t}}$

e.g., already have data on $y_{2t},\ldots,y_{nt}$ and ask how a 1-unit change in y_{1t} affects forecast.

Could instead ask $\dfrac{\partial E(\mathbf{y}_{t+s}|y_{1t},\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial y_{1t}}$

e.g., don't have any data from period t except for y_{1t} and ask how 1-unit change in y_{1t} affects forecast.

Knowing y_{1t} gives us information about $y_{2t},\ldots,y_{nt}$ if VAR forecast errors are correlated.

How calculate $\underset{(n\times 1)}{\mathbf{h}_{s1}} = \dfrac{\partial E(\mathbf{y}_{t+s}|y_{1t},\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial y_{1t}}$?

Method 1: local projection

Estimate by n OLS equations

$$\mathbf{y}_{t+s} = \mathbf{c}_s + \mathbf{h}_{s1} y_{1t} + \mathbf{H}_{s2}\mathbf{y}_{t-1} + \cdots + \mathbf{H}_{sp}\mathbf{y}_{t-p+1} + \mathbf{u}_{t+s}$$

Method 2: calculate answer implied by VAR

$$\underset{(n\times 1)}{\mathbf{y}_t} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$$

$$\underset{(k\times 1)}{\mathbf{x}_t} = (1, \mathbf{y}'_{t-1}, \mathbf{y}'_{t-2}, \ldots, \mathbf{y}'_{t-p})'$$

$$k = np + 1$$

$$\mathbf{y}_t = \mathbf{\Gamma}'\mathbf{x}_t + \boldsymbol{\varepsilon}_t$$

$$E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t) = \mathbf{\Omega}$$

Given parameters, observation of $y_{1t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}$ allows us to observe

$$\varepsilon_{1t} = y_{1t} - \boldsymbol{\gamma}_1' \mathbf{x}_t$$

Can calculate optimal forecast of ε_{it} given ε_{1t} as

$$E(\varepsilon_{it}|\varepsilon_{1t}) = \frac{\sigma_{i1}}{\sigma_{11}}\varepsilon_{1t}$$

$$E(\boldsymbol{\varepsilon}_t|\varepsilon_{1t}) = \begin{bmatrix} 1 \\ \sigma_{21}/\sigma_{11} \\ \vdots \\ \sigma_{n1}/\sigma_{11} \end{bmatrix}\varepsilon_{1t} = \mathbf{a}_1\varepsilon_{1t}$$

$$\frac{\partial E(\mathbf{y}_t | y_{1t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{1t}} = \mathbf{a}_1$$

$$\frac{\partial E(\mathbf{y}_{t+s} | y_{1t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{1t}} = \frac{\partial E(\mathbf{y}_{t+s} | \mathbf{y}_t, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial \mathbf{y}_t'} \frac{\partial E(\mathbf{y}_t | y_{1t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{1t}}$$

$$= \mathbf{\Psi}_s \mathbf{a}_1$$

$$\underset{(n\times k)}{\hat{\boldsymbol{\Gamma}}'} = \left(\sum_{t=1}^{T} \mathbf{y}_t \mathbf{x}_t'\right)\left(\sum_{t=1}^{T} \mathbf{x}_t \mathbf{x}_t'\right)^{-1} \Rightarrow \hat{\boldsymbol{\Psi}}_s$$

$$\underset{(n\times 1)}{\hat{\boldsymbol{\varepsilon}}_t} = \mathbf{y}_t - \hat{\boldsymbol{\Gamma}}'\mathbf{x}_t$$

$$\underset{(n\times n)}{\hat{\boldsymbol{\Omega}}} = T^{-1}\sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_t \hat{\boldsymbol{\varepsilon}}_t' \Rightarrow \hat{\mathbf{a}}_1 = \begin{bmatrix} 1 \\ \hat{\sigma}_{21}/\hat{\sigma}_{11} \\ \vdots \\ \hat{\sigma}_{n1}/\hat{\sigma}_{11} \end{bmatrix}$$

Could also do this using Cholesky factor:

$\hat{\mathbf{\Omega}} = \hat{\mathbf{P}}\hat{\mathbf{P}}'$ ($\hat{\mathbf{P}}$ lower triangular)

$\hat{\mathbf{a}}_1 = \hat{\mathbf{p}}_1/\hat{p}_{11}$

$\hat{\mathbf{p}}_1$ = column 1 of $\hat{\mathbf{P}}$

$\hat{p}_{11}$ = row 1 col 1 element of $\hat{\mathbf{P}}$

$$\boldsymbol{\Psi}_s \mathbf{a}_1 = \frac{\partial E(\mathbf{y}_{t+s} | y_{1t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{1t}}$$

is effect of one-unit increase in y_{1t} or ε_{1t} on forecast

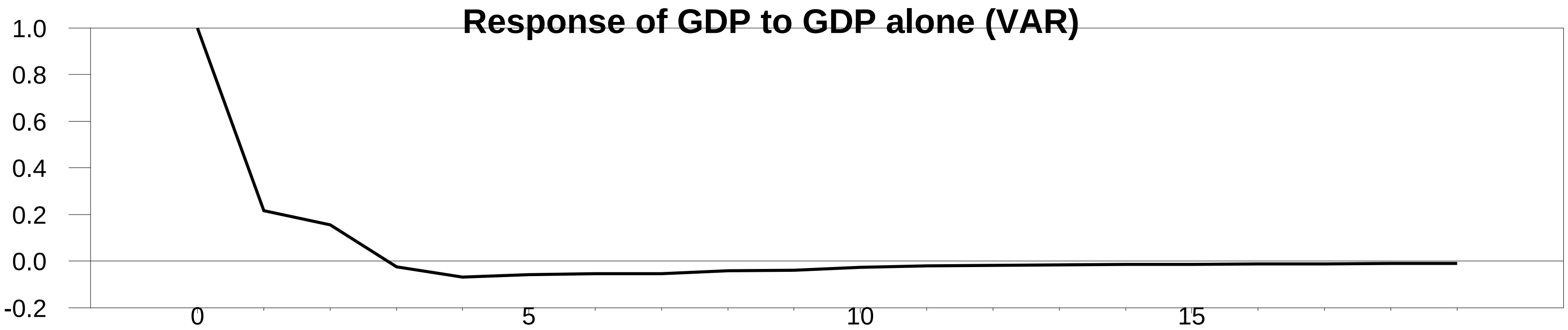
$\boldsymbol{\Psi}_s \mathbf{p}_1$ is effect of one-standard-deviation increase in ε_{1t} on forecast.

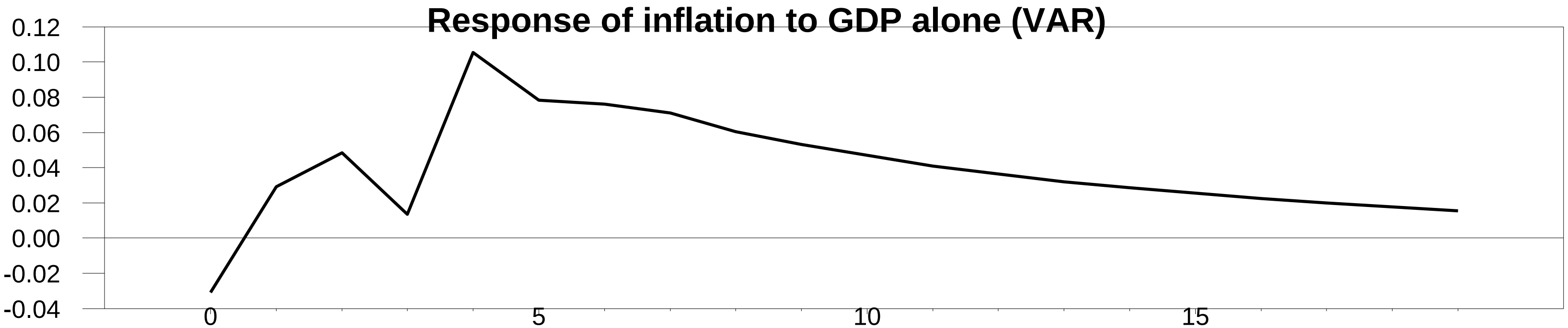
$\boldsymbol{\Psi}_s \mathbf{a}_1$ shows IRF to shock in observed units

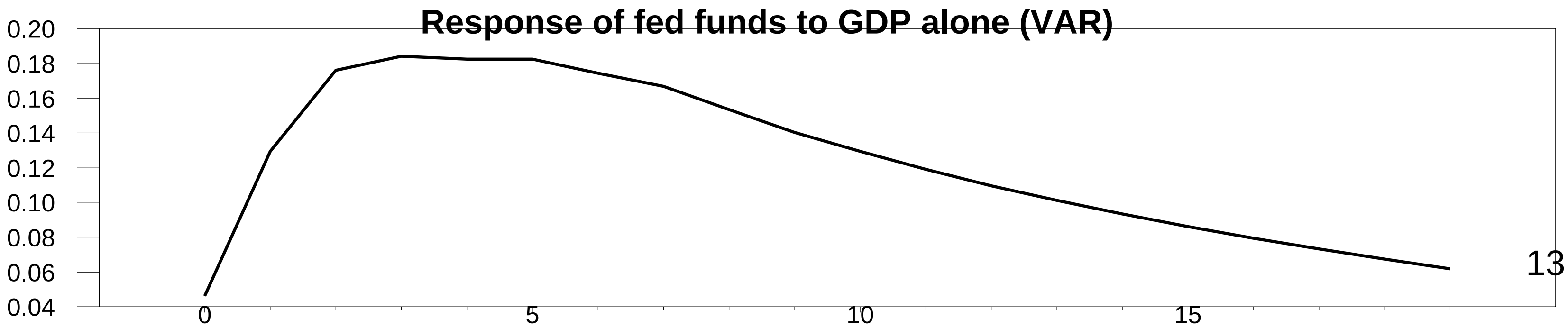
$\boldsymbol{\Psi}_s \mathbf{p}_1$ shows IRF to shock of typical size

Plots will look identical just with different units on vertical axis $\boldsymbol{\Psi}_s \mathbf{p}_1 = \boldsymbol{\Psi}_s \mathbf{a}_1 p_{11}$

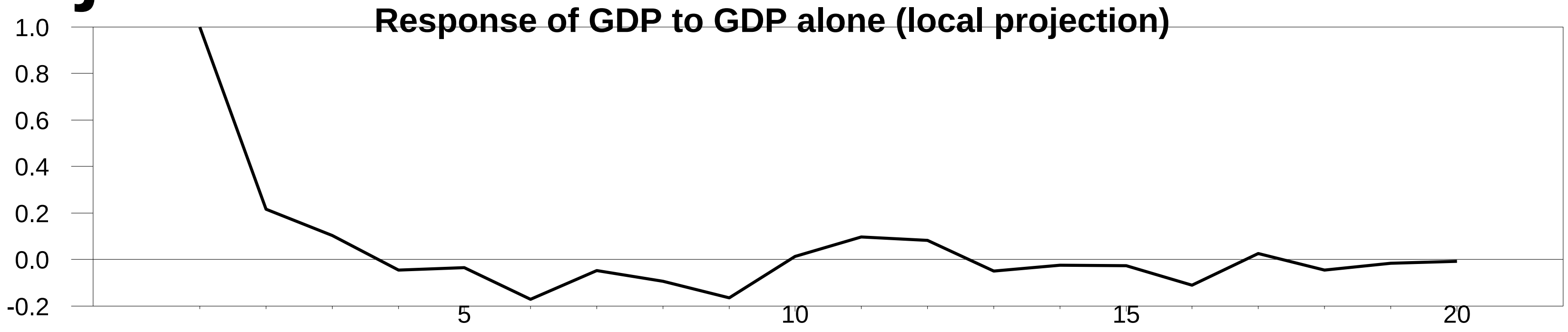
Recursively orthgonalized VAR estimated 1954-2007

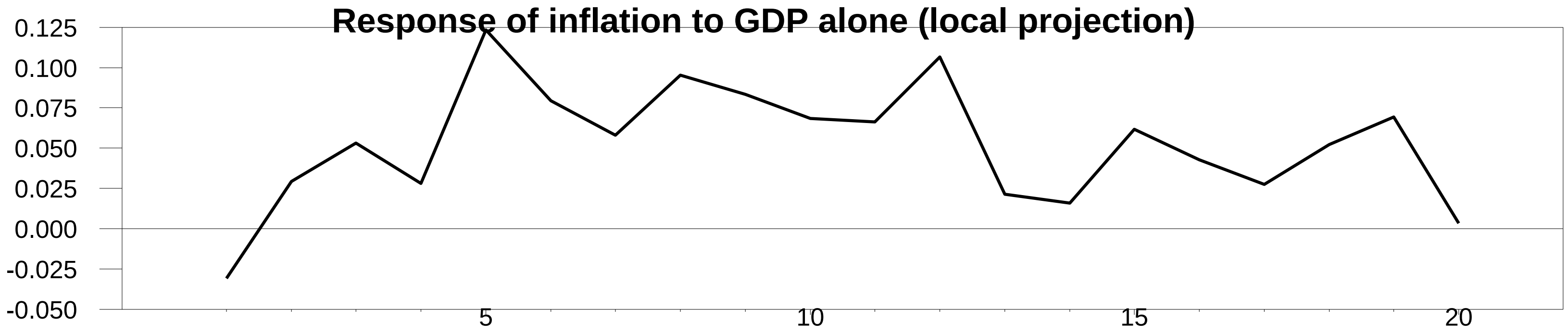


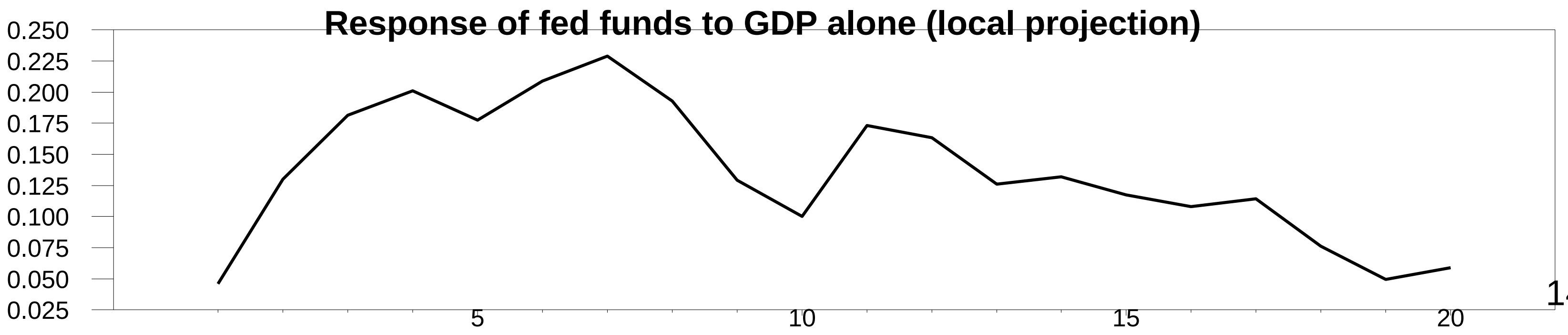




Recursively orthgonalized local projection estimated 1954-2007







Suppose next that we've observed y_{1t} and y_{2t} but not $y_{3t}, y_{4t}, \ldots, y_{nt}$.

What is effect on forecast of changing y_{2t}?

$$\frac{\partial E(\mathbf{y}_t | y_{1t}, y_{2t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{2t}} = \mathbf{a}_2 = \mathbf{p}_2 / p_{22}$$

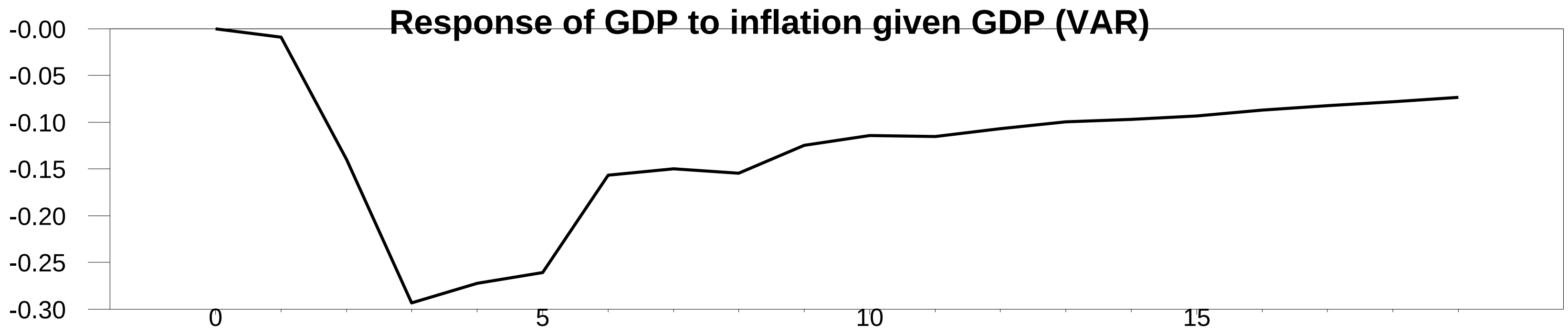
$\mathbf{p}_2$ = column 2 of $\mathbf{P}$

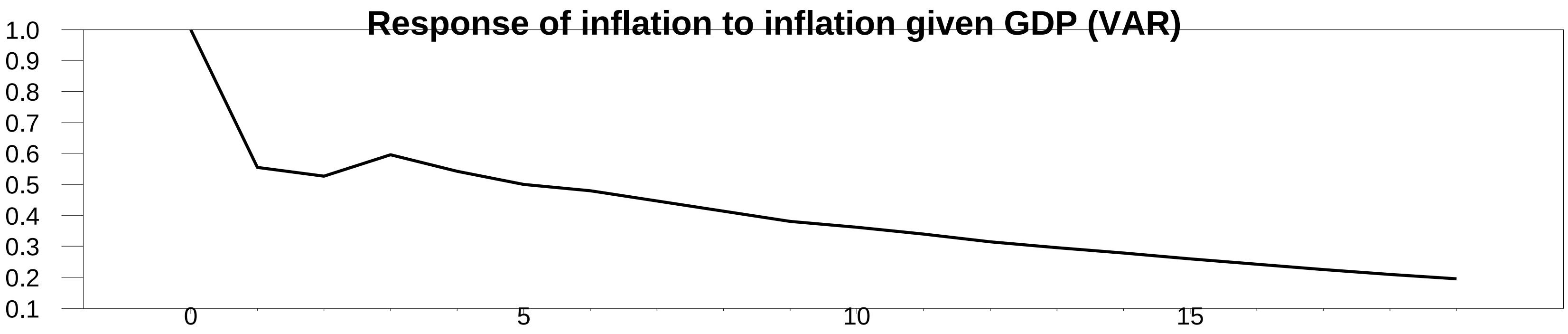
p_{22} = row 2 col 2 element of $\mathbf{P}$

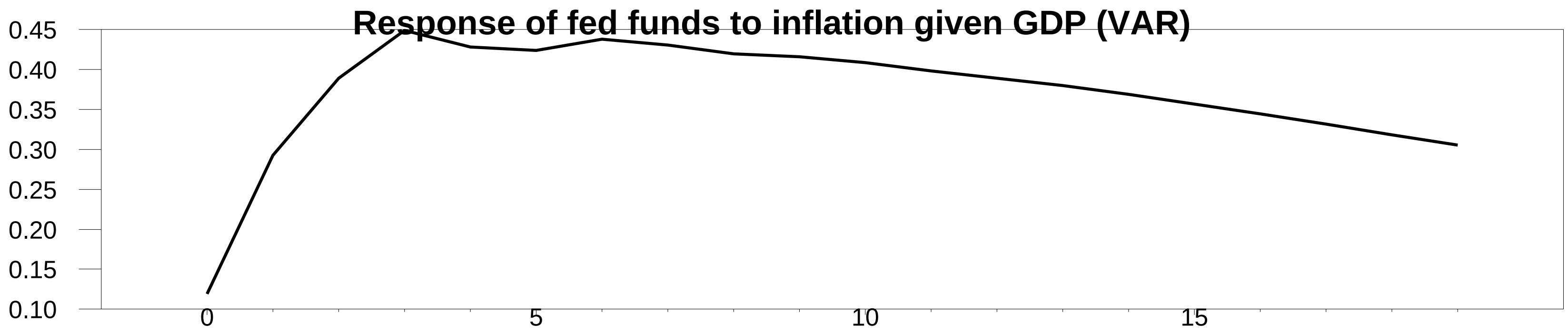
first element of $\mathbf{a}_2$ is zero

$$\frac{\partial E(\mathbf{y}_{t+s}|y_{1t},y_{2t},\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial y_{2t}} = \frac{\partial E(\mathbf{y}_{t+s}|\mathbf{y}_t,\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial \mathbf{y}_t'} \frac{\partial E(\mathbf{y}_t|y_{1t},y_{2t},\mathbf{y}_{t-1},\ldots,\mathbf{y}_{t-p})}{\partial y_{2t}}$$

$$= \boldsymbol{\Psi}_s \mathbf{a}_2$$


Response of GDP to inflation given GDP (VAR)
-0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
0
5
10
15


Response of inflation to inflation given GDP (VAR)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15


Response of fed funds to inflation given GDP (VAR)
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0
5
10
15

$(n \times n)$ matrix of recursively orthogonalized shocks:

$\boldsymbol{\Psi}_s \mathbf{P}$ or $\boldsymbol{\Psi}_s \mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ a_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} = \mathbf{P}[\text{diag}(\mathbf{P})]^{-1}$$

Note last col of $\boldsymbol{\Psi}_s \mathbf{A}$ is identical to last col of $\boldsymbol{\Psi}_s$

We have broken down the news arriving in period t into n separate uncorrelated components

$\varepsilon_{1t} = y_{1t} - \boldsymbol{\gamma}_1' \mathbf{x}_t$ = news about y_{1t}

$u_{2t} = \varepsilon_{2t} - a_{21}\varepsilon_{1t}$ = news about y_{2t} not already revealed by y_{1t}

$$\vdots$$

$u_{nt} = \varepsilon_{nt} - E(\varepsilon_{nt} | \varepsilon_{1t}, \ldots, \varepsilon_{n-1,t})$

= news about y_{nt} not already revealed by $y_{1t}, \ldots, y_{n-1,t}$

Simple way to summarize these components:

$$\underset{(n\times 1)}{\mathbf{v}_t} = \mathbf{P}^{-1}\boldsymbol{\varepsilon}_t = \begin{bmatrix} p^{11} & 0 & \cdots & 0 \\ p^{21} & p^{22} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ p^{n1} & p^{n2} & \cdots & p^{nn} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \vdots \\ \varepsilon_{nt} \end{bmatrix}$$

$$E(\mathbf{v}_t\mathbf{v}_t') = \mathbf{P}^{-1}E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')\mathbf{P}'^{-1} = \mathbf{P}^{-1}\boldsymbol{\Omega}\mathbf{P}'^{-1}$$

$$= \mathbf{P}^{-1}\mathbf{P}\mathbf{P}'\mathbf{P}'^{-1} = \mathbf{I}_n$$

$\mathbf{v}_t$ is a linear combination of $\boldsymbol{\varepsilon}_t$ whose elements are uncorrelated with each other

$$\begin{aligned}
v_{1t} &= p^{11}\varepsilon_{1t} \\
&= \text{rescaled error forecasting } y_{1t} \\
v_{2t} &= p^{21}\varepsilon_{1t} + p^{22}\varepsilon_{2t} \\
&= \text{rescaled error forecasting } \varepsilon_{2t} \text{ from } \varepsilon_{1t} \\
v_{nt} &= p^{n1}\varepsilon_{1t} + p^{n2}\varepsilon_{2t} + \cdots + p^{nn}\varepsilon_{nt} \\
&= \text{rescaled error forecasting } \varepsilon_{nt} \text{ from } \varepsilon_{1t}, \ldots, \varepsilon_{n-1,t}
\end{aligned}$$

B. Variance decomposition

$$\mathbf{y}_{t+s} = \hat{\mathbf{y}}_{t+s|t} + \mathbf{\Psi}_0 \boldsymbol{\varepsilon}_{t+s} + \mathbf{\Psi}_1 \boldsymbol{\varepsilon}_{t+s-1} + \mathbf{\Psi}_2 \boldsymbol{\varepsilon}_{t+s-2} + \cdots + \mathbf{\Psi}_{s-1} \boldsymbol{\varepsilon}_{t+1}$$

$$E(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t})(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t}) = \sum_{m=0}^{s-1} \mathbf{\Psi}_m \mathbf{\Omega} \mathbf{\Psi}_m'$$

$$\boldsymbol{\varepsilon}_t = \mathbf{P}\mathbf{v}_t = \mathbf{p}_1 v_{1t} + \mathbf{p}_2 v_{2t} + \cdots + \mathbf{p}_n v_{nt}$$

Contribution of $v_{i,t+1}, v_{i,t+2}, \ldots, v_{i,t+s}$ to forecast error:

$$\mathbf{\Psi}_0 \mathbf{p}_i v_{i,t+s} + \mathbf{\Psi}_1 \mathbf{p}_i v_{i,t+s-1} + \cdots + \mathbf{\Psi}_{s-1} \mathbf{p}_i v_{i,t+1}$$

$$E(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t})(\mathbf{y}_{t+s} - \hat{\mathbf{y}}_{t+s|t})$$

$$= \sum_{m=0}^{s-1} \mathbf{\Psi}_m \mathbf{p}_1 \mathbf{p}_1' \mathbf{\Psi}_m' + \cdots + \sum_{m=0}^{s-1} \mathbf{\Psi}_m \mathbf{p}_n \mathbf{p}_n' \mathbf{\Psi}_m'$$

First term: amount by which could reduce MSE if we knew the values of $\varepsilon_{1,t+1}, \ldots, \varepsilon_{1,t+s}$

Second term: amount by which we could reduce MSE if we knew the values of $u_{2,t+1}, \ldots, u_{2,t+s}$

Decomposition of Variance for Series GDPCH

Step	Std Error	GDPCH	INFLATION	FEDFUNDS
1	3.08741913	100.000	0.000	0.000
2	3.15963628	99.994	0.001	0.005
3	3.36710884	90.098	0.218	9.684
4	3.41017407	87.881	1.147	10.972
5	3.43030191	87.226	1.928	10.846
6	3.45041262	86.487	2.627	10.886
7	3.45981410	86.245	2.871	10.883
8	3.46834423	86.053	3.093	10.854
9	3.47514728	85.850	3.331	10.819
10	3.47997093	85.727	3.483	10.789
11	3.48341375	85.612	3.612	10.776
12	3.48663478	85.487	3.744	10.769
13	3.48920625	85.388	3.856	10.756
14	3.49137054	85.302	3.954	10.745
15	3.49341827	85.216	4.047	10.737
16	3.49526789	85.139	4.132	10.729
17	3.49686734	85.072	4.206	10.721
18	3.49830813	85.011	4.273	10.716
19	3.49960777	84.956	4.333	10.711
20	3.50074296	84.907	4.385	10.708

Decomposition of Variance for Series INFLATION

Step	Std Error	GDPCH	INFLATION	FEDFUNDS
1	1.12790671	0.711	99.289	0.000
2	1.30103516	1.009	97.570	1.421
3	1.43945588	1.909	96.580	1.511
4	1.58901603	1.634	97.023	1.343
5	1.73329210	4.906	93.947	1.147
6	1.83842144	6.087	92.878	1.034
7	1.93358191	6.981	91.717	1.302
8	2.01323981	7.629	90.802	1.569
9	2.07617743	7.983	90.376	1.641
10	2.12893320	8.188	89.988	1.824
11	2.17571089	8.286	89.663	2.051
12	2.21501613	8.322	89.481	2.197
13	2.24826561	8.329	89.336	2.335
14	2.27737512	8.303	89.209	2.488
15	2.30262070	8.268	89.109	2.623
16	2.32420833	8.229	89.031	2.740
17	2.34293779	8.186	88.958	2.856
18	2.35921378	8.141	88.891	2.968
19	2.37321184	8.097	88.834	3.069
20	2.38526891	8.056	88.782	3.163

Decomposition of Variance for Series FEDFUNDS

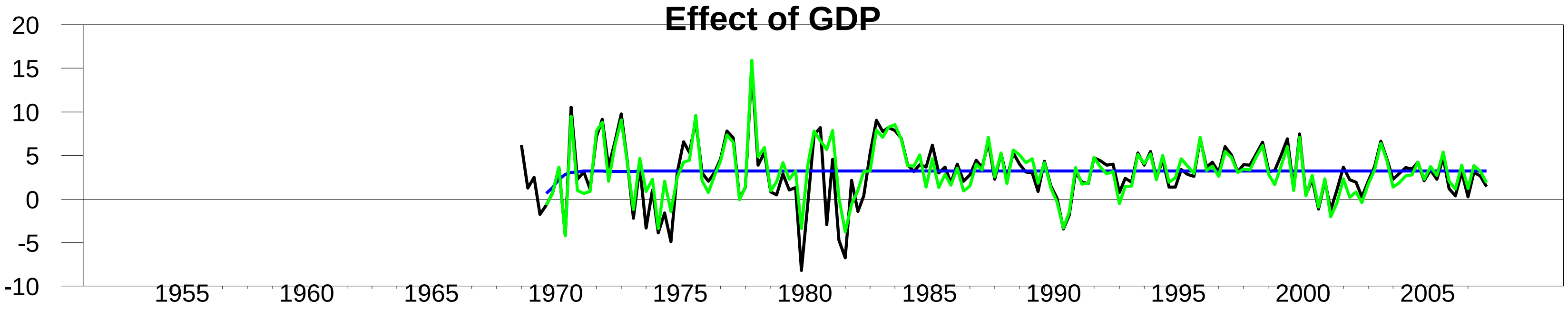
Step	Std Error	GDPCH	INFLATION	FEDFUNDS
1	0.78294284	3.309	2.939	93.752
2	1.24088336	11.710	8.195	80.095
3	1.51928430	20.625	13.752	65.623
4	1.76776027	25.568	18.317	56.116
5	1.97194315	28.715	20.684	50.601
6	2.13303190	31.520	22.672	45.807
7	2.27057752	33.433	24.707	41.860
8	2.39054533	34.802	26.385	38.813
9	2.49021257	35.694	27.902	36.404
10	2.57500835	36.215	29.393	34.392
11	2.64925525	36.494	30.775	32.732
12	2.71414804	36.608	32.042	31.350
13	2.77123546	36.605	33.230	30.165
14	2.82208295	36.523	34.335	29.142
15	2.86734779	36.391	35.348	28.261
16	2.90764045	36.226	36.275	27.499
17	2.94364156	36.040	37.125	26.836
18	2.97580221	35.843	37.897	26.260
19	3.00448248	35.642	38.597	25.761
20	3.03006847	35.441	39.230	25.329

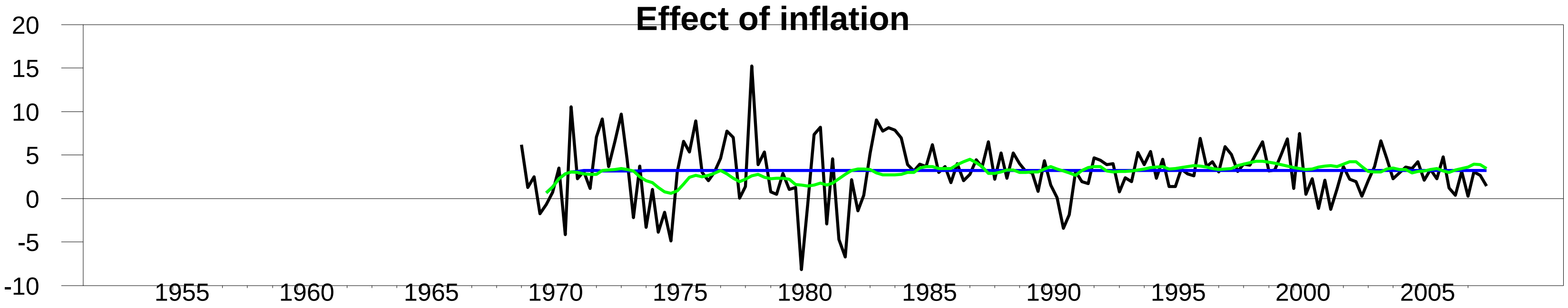
C. Historical decomposition

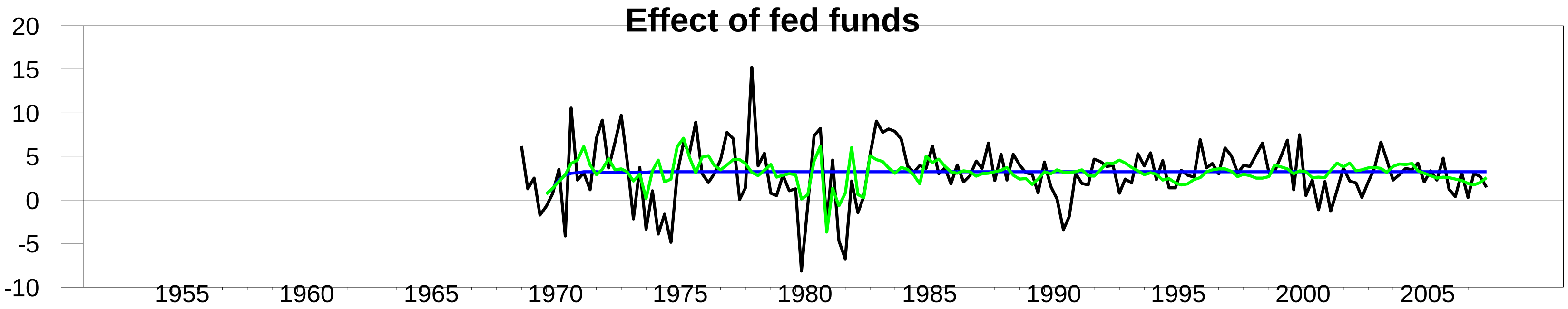
$$\begin{aligned}\mathbf{y}_{t+s} &= \hat{\mathbf{y}}_{t+s|t} + \sum\nolimits_{m=0}^{s-1} \boldsymbol{\Psi}_m \boldsymbol{\varepsilon}_{t+s-m} \\ &= \hat{\mathbf{y}}_{t+s|t} + \sum\nolimits_{m=0}^{s-1} \boldsymbol{\Psi}_m \left[\mathbf{p}_1 v_{1,t+s-m} + \cdots + \mathbf{p}_n v_{n,t+s-m}\right]\end{aligned}$$

Can decompose the observed value for any variable at any date into component that could have been predicted as of some earlier date plus innovations in individual $v_{i,t+m}$ since then.

Historical Decomposition of GDP







D. Structural interpretation

Suppose we hypothesized the following

structural model for the behavior of the Fed:

$$i_t = \lambda_3 + \psi_y y_t + \psi_\pi \pi_t + \mathbf{b}'_{31}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{3p}\mathbf{y}_{t-p} + u_{3t}$$

i_t = fed funds rate

y_t = GDP growth rate

π_t = inflation rate

ψ_y, ψ_π = coefficients in Taylor Rule

$\mathbf{b}_{3m}$ allow for inertia in monetary policy

u_{3t} = serially uncorrelated shock to monetary policy

= deviation from Fed's usual rule, uncorrelated

with $\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}$ by definition

Would like to know $\partial \mathbf{y}_{t+s} / \partial u_{3t}$

Suppose I also thought there was a Phillips Curve of the form

$$\pi_t = \lambda_2 + \alpha y_t + \mathbf{b}_{21}'\mathbf{y}_{t-1} + \cdots + \mathbf{b}_{2p}'\mathbf{y}_{t-p} + u_{2t}$$

$\alpha =$ slope of Phillips Curve

$\mathbf{b}_{2m}$ allow for inertia in PC

$u_{2t} =$ unpredictable shock to PC

u_{2t} uncorrelated with $\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}$ by definition

u_{2t} also assumed to be uncorrelated with u_{3t}

(assumption that monetary policy shocks take more than one period to affect inflation)

Model equilibrium output as

$$y_t = \lambda_1 + \mathbf{b}'_{11}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{1p}\mathbf{y}_{t-p} + u_{1t}$$

u_{1t} = error forecasting GDP one period ahead

u_{1t} uncorrelated with $\mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p}$ by definition

u_{1t} also assumed to be uncorrelated with u_{2t}, u_{3t}

(assumption that PC and monetary shocks take more than one period to affect output)

$$i_t = \lambda_3 + \psi_y y_t + \psi_\pi \pi_t + \mathbf{b}_{31}' \mathbf{y}_{t-1} + \cdots + \mathbf{b}_{3p}' \mathbf{y}_{t-p} + u_{3t}$$

Above assumptions mean u_{3t} uncorrelated with y_t and π_t.

$\Rightarrow$ could estimate by OLS

$\hat{\psi}_y$ and $\hat{\psi}_\pi$ are same as step 0 Jordá projection

$\hat{\psi}_y$ and $\hat{\psi}_\pi$ are same as $\hat{a}_{31}$ and $\hat{a}_{32}$

$$\pi_t = \lambda_2 + \alpha y_t + \mathbf{b}'_{21}\mathbf{y}_{t-1} + \cdots + \mathbf{b}'_{2p}\mathbf{y}_{t-p} + u_{2t}$$

Above assumptions mean u_{2t} uncorrelated with y_t.

$\Rightarrow$ could estimate by OLS

$\hat{\alpha}$ is same as step 0 Jordá projection

$\hat{\alpha}$ is same as $\hat{a}_{21}$

Conclusion: under above assumptions with

$$\mathbf{A} = \mathbf{P}[\text{diag}(\mathbf{P})]^{-1}$$

$$\mathbf{u}_t = \mathbf{A}^{-1}\boldsymbol{\varepsilon}_t$$

$$u_{1t} = \varepsilon_{1t}$$

The error I make forecasting y_{1t} given $\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p}$ is the shock to equilibrium output.

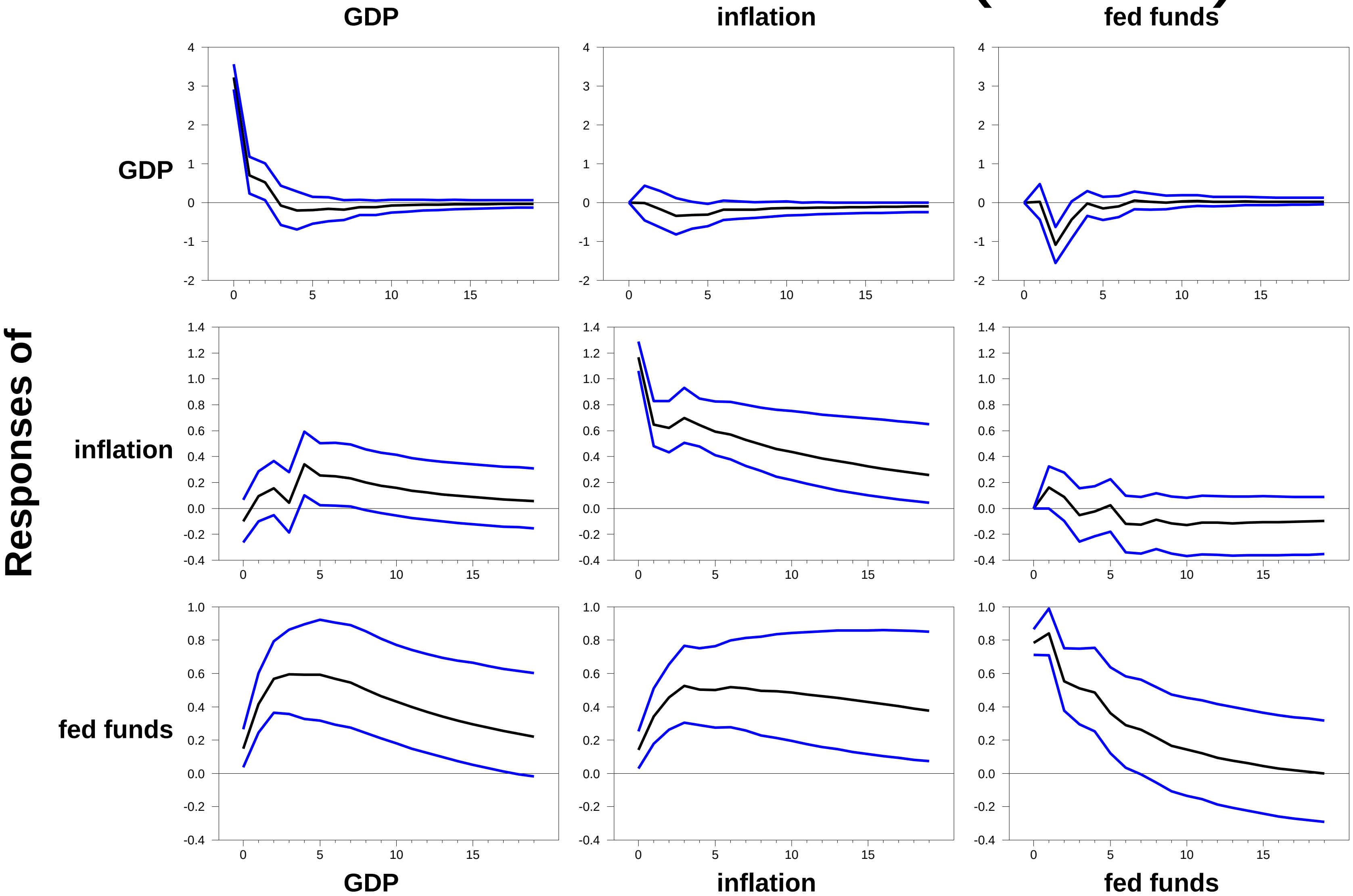
The error I make forecasting y_{2t} given $y_{1t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p}$ is the shock to PC.

The error I make forecasting y_{3t} given $y_{1t}, y_{2t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p}$ is the shock to monetary policy.

$$\frac{\partial E(\mathbf{y}_{t+s} | y_{1t}, y_{2t}, y_{3t}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{3t}} = \frac{\partial \mathbf{y}_{t+s}}{\partial u_{3t}}$$

Recursively orthogonalized VAR gives the dynamic effects of monetary policy.

Orthogonal Cholesky IRF with 95% confidence intervals (54-07)



- A monetary contraction (higher fed funds rate) is followed by slower GDP growth 2-3 quarters later
- But unanticipated monetary policy shocks account for only 10% of variance of output
- Most of variation in fed funds rate comes from predictable response of monetary policy to output and inflation
- A monetary contraction is followed by higher inflation (known as “price puzzle”)

- Assumption-free statement of price puzzle:
  - if you tell me that fed funds rate is higher than I would have predicted given current output, inflation, and lags, then I will revise my expectation of future inflation up.
- Natural interpretation:
  - Fed raised funds rate because it anticipated future inflation.
  - Our 3-variable equation is too simplistic a description of Fed

- Popular “fix” for price puzzle:
  - Add other variables that better capture information about future inflation (such as commodity prices) to Fed policy equation

Christiano, Eichenbaum, Evans (1996)

y_{1t} = log of real GDP

y_{2t} = log of GDP deflator

y_{3t} = index of sensitive commodity prices

y_{4t} = fed funds rate

y_{5t} = nonborrowed reserves

y_{6t} = total reserves

y_{7t} = one of a set of macro variables

Structural model:

$$\mathbf{B}_0\mathbf{y}_t = \mathbf{B}\mathbf{x}_t + \mathbf{u}_t$$

$$\mathbf{x}_t = (1, \mathbf{y}_{t-1}^{'}, \mathbf{y}_{t-2}^{'}, \ldots, \mathbf{y}_{t-p}^{'})'$$

$\mathbf{u}_t =$ vector of structural shocks

$E(\mathbf{u}_t\mathbf{u}_t') = \mathbf{D}$ (diagonal)

$$\mathbf{B}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ x & 1 & 0 & 0 & 0 & 0 & 0 \\ x & x & 1 & 0 & 0 & 0 & 0 \\ x & x & x & 1 & 0 & 0 & 0 \\ x & x & x & x & 1 & 0 & 0 \\ x & x & x & x & x & 1 & 0 \\ x & x & x & x & x & x & 1 \end{bmatrix}$$

Variable 4 is fed funds rate, equation 4 is monetary policy equation.

Note that

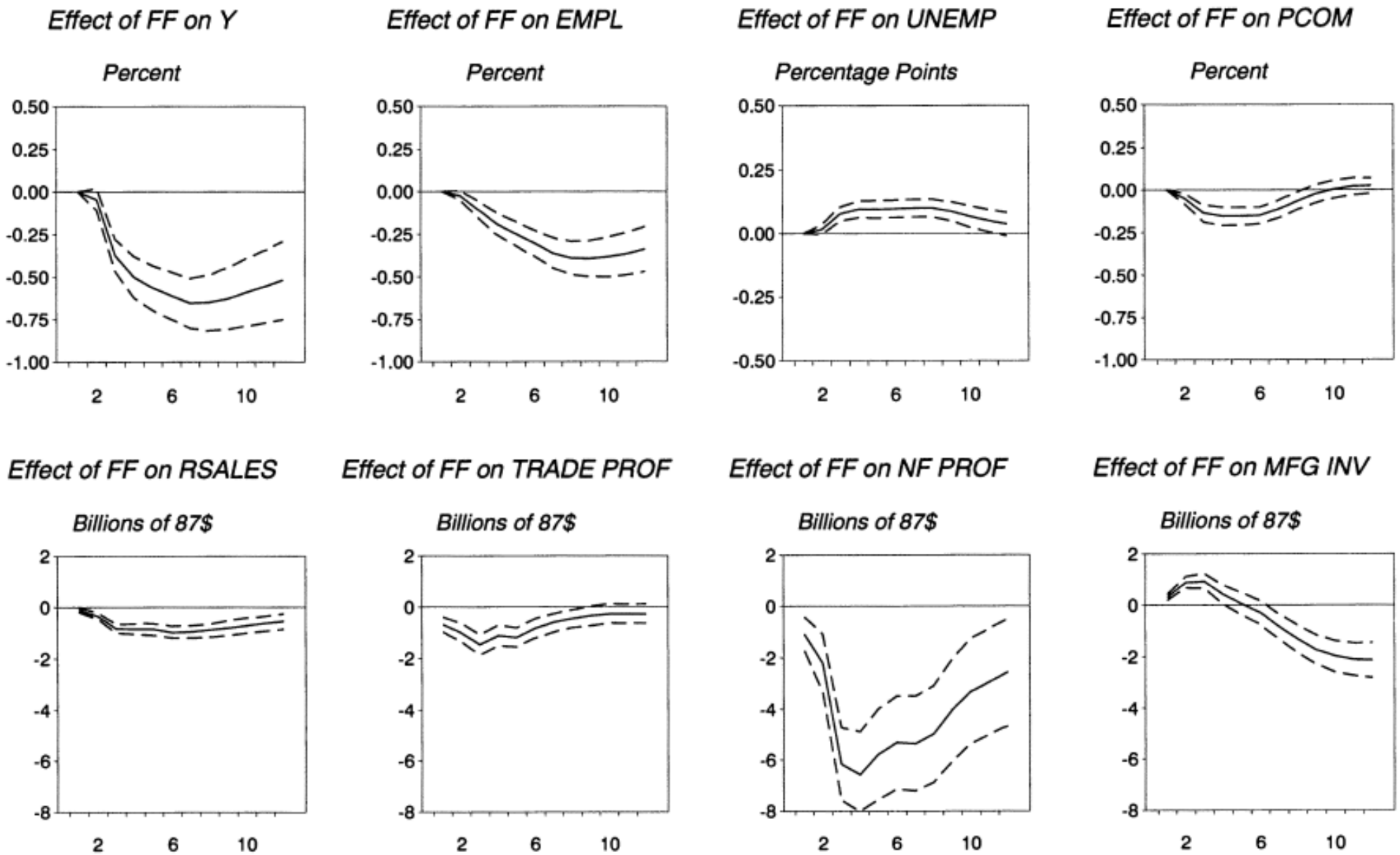
$$\frac{\partial E(\mathbf{y}_{t+s}|y_{1t}, y_{2t}, y_{3t}, y_{4t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p})}{\partial y_{4t}} = \frac{\partial E(\mathbf{y}_{t+s}|y_{2t}, y_{1t}, y_{3t}, y_{4t}, \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \ldots, \mathbf{y}_{t-p})}{\partial y_{4t}}$$

Will have the identical answer for effect of variable 4 any way we order variables 1-3 and any way we order variables 5-7.

Jordá estimate identical if reorder (keeping 4 in place).

If all we care about is effect of monetary policy, we only need to assume block-recursive

$$\mathbf{B}_0 = \begin{bmatrix} x & x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 & 0 \\ x & x & x & 1 & 0 & 0 & 0 \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \\ x & x & x & x & x & x & x \end{bmatrix}$$



67% confidence bands

E. Generalized IRFs

- If we put fed funds fourth, estimated effect of monetary policy does not depend on how we order variables 1-3.
- But if we switch fed funds from 4 to 3, results could change

- Pesaran and Shin (1998): “generalized impulse-response function”
  - Put variable #1 first to find effect of variable 1
  - Put variable #2 first to find effect of variable 2
  - Put variable #n first to find effect of variable n

GIRF: for every i, calculate

$$\frac{\partial E(\mathbf{y}_{t+s} | y_{it}, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_{t-p})}{\partial y_{it}}$$

- Conclusion: any IRF or GIRF is giving answer to a forecasting question.
- Best practice: describe forecasting question explicitly and explain the reason that question is interesting.